



This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

### Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

- + *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
- + *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
- + *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
- + *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

### About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at <http://books.google.com/>

**HARVARD UNIVERSITY**



**LIBRARY OF THE  
GRADUATE SCHOOL  
OF EDUCATION**





# A PRIMER OF NUMBER

A TEACHERS' MANUAL FOR  
FIRST AND SECOND GRADES

By

FRANK RIGLER

FORMERLY SUPERINTENDENT OF CITY SCHOOLS  
PORTLAND, OREGON

REVISED EDITION



CHICAGO

O. P. BARNES, PUBLISHER

HARVARD UNIVERSITY  
GRADUATE SCHOOL OF EDUCATION  
MEREDITH S. CUTMAN LIBRARY

LIBRARY OF THE  
GRADUATE SCHOOL OF EDUCATION  
1930

QA135  
.R5

B

---

---

Copyright, 1913,  
By O. P. BARNES

---

---

## PREFACE

Most methods of teaching primary number make the purpose of the first year's work the memorizing of the so-called forty-five facts of addition and subtraction. This is dry and difficult work for young children, and results in such a dislike for the subject that in not a few places number work has been discarded from the course of study of the first school year. Some go so far as to say that children should not be given concrete problems to solve earlier than the fourth year of school, because they are unable to do the reasoning required for the solution of such problems. If concrete problems are to be solved by *verbal* reasoning the argument is not without force; but the writer believes that verbal reasoning has no place in the early solution of arithmetical problems.

Six-year-old children like to count. This impulse should be used as the means of developing their number sense. Counting is the fundamental operation; all computation is derived from it, and may be made ultimately to displace it, the remembered *modes of counting* being finally used only to determine by what operation the problem under consideration is to be solved.

The first year's number work then, consists in counting objects grouped in various ways, and in making numerical records of such groupings. Children delight in doing this, and achieve such results as sometimes seem incomprehensible to those not familiar with the method. It is believed, however, that a careful reading of this book will convince primary teachers that the method follows the child's natural development in number power, and that the means employed are pedagogically sound.

PORTLAND, OREGON.  
*July, 1913.*

FRANK RIGLER.





# A PRIMER OF NUMBER

---

## PLURALITY AND NUMBER DISTINGUISHED

A group of similar objects, (say marbles), possesses the attribute of plurality. We may say of it, "There are a *few* marbles," or "There are *several* marbles," or "There are *many* marbles."

A second group of the same kind of objects may differ from the first group in its degree of plurality; yet we may be able to make the same assertions of it that we made of the first group.

Each degree of plurality has a particular name, the lowest degree being called "two," the next "three," the next "four," etc. These names, when memorized in their proper order, constitute a mental scale by which the degree of plurality of any group of separated objects may be measured, (i. e., ascertained).

The process of measurement is called "counting," and consists of a "one-to-one" application of the words of the scale to the objects of the group. The word of the scale applied to the last object of the group names the group's degree of plurality. (See "To Teach Counting," page 6.)

Our *notion* of the group before counting is called *plurality*; our *notion* of it after counting is called *number*.

Therefore, *Number is measured plurality*.

## I. TO TEACH COUNTING

When children enter school they have some comprehension of degree in plurality and some knowledge of the names of different degrees. This knowledge needs to be clarified, fixed, and extended. The following points require attention:

*First.* A new number name, as "six," should always be given in the presence of a group of objects of the degree of plurality indicated by the number.

*Second.* What is commonly called "counting without objects," i. e., uttering the words "one," "two," "three," etc., which name the different degrees of plurality, is a useful exercise, provided that no new number words are presented in this way. The teacher should understand, of course, that this is not counting, but is simply making pupils ready and facile with the word series which constitutes the scale used in measuring degrees of plurality.

*Third.* The errors of pupils arise from two causes: (a) imperfect knowledge of the word series "one," "two," "three," etc.; (b) failure to make a "one-to-one" application of this word series to the group of objects counted. The erroneous application may be either "one (word) to *two* (objects)," as when

the pupil touches two objects while saying "three," or it may be "two (words) to *one* (object)," as when the pupil utters the words "two," "three," while touching one of the objects counted.

### CORRECT COUNTING

(Consisting of a one-to-one application of the words of the scale to the objects counted)

WORDS OF SCALE:	"One";	"Two";	"Three";	"Four";	"Five".
OBJECTS COUNTED:	○	○	○	○	○

### ERRORS IN COUNTING

*First. The pupil does not know the scale.*

WORDS OF SCALE:	"One";	"Two";	"Three"	"Four"	"Six"
OBJECTS COUNTED:	○	○	○	○	○

*Third. One word is applied to two objects.*

WORDS OF SCALE:	"One";	"Two";	"Three"	"Four"
OBJECTS COUNTED:	○	○	○-○	○

*Second. Two words are applied to one object.*

WORDS OF SCALE:	"One";	"Two-three";	"Four"	"Five";	"Six".
OBJECTS COUNTED:	○	○	○	○	○

*Fourth.* Careful distinction should be made between the following two general modes of counting:

(1) Ascertaining the degree of plurality of a given group.

(2) Constructing a group of a given degree of plurality.

This distinction will be made clear by the following simple examples:

(a) Teacher hands the pupil five pencils and asks him how many she has given him. Pupil counts them and answers, "Five."

Notice that the teacher constructed the group, and the pupil ascertained its degree of plurality.

(b) Teacher sends the pupil to a box of pencils with instructions to bring her five.

Notice that the degree of plurality is here given the pupil, and he constructs a group possessing that degree.

The latter mode of counting is more difficult than the former, because in the former the pupil has simply to apply the word series to the constructed group of objects, while in the latter mode he must, at the same time he is making this application, keep in mind the word of the series with which he must stop.

Fifth. The material upon which counting is practiced should be carefully graded, as follows:

(1) Count objects both seen and touched.

(2) Count objects seen but not touched.

(3) Count sounds, as how many taps are made by some one not seen, or how many sounds in a slowly uttered word.

(4) Count repetitions of an act, as how many times a pupil crosses the room.

The following exercises will show how each of the two general modes of counting may be practiced upon the material just described.

(1) *Objects both seen and touched*

(a) Teacher: "How many books are on the table?"

Pupil counts the books and answers, "Seven."

(b) Teacher: "John, you may take five books from the shelf and put them on the table."

Pupil does as directed.

(2) *Objects seen but not touched*

A row of blocks is upon the number table.

(a) Teacher says, "How many blocks are on the table?"

Pupil counts without touching and answers, "Eight."

(b) Teacher says, "Move five blocks at one time from the row."

Pupil counts with the eye up to the fifth block and moves the five blocks.

(3) *Sounds*

(a) Teacher says, "Count how many times I tap on the table."

Teacher taps; pupil counts and answers, "Seven."

(b) Teacher says, "Willie, you may tap nine times on the table."

Willie does so.

#### (4) *Repetition of an act*

(a) Teacher says, "Count how many times I raise my hand."

Teacher raises hand repeatedly. Pupil counts and answers, "Nine."

(b) Teacher says, "George, you may cross the room six times."

George does so.

## II. WRITING NUMBERS

When pupils can count to six or seven, instruction in writing numbers should be taken up. After this the two lines of work may be carried on concurrently, counting being somewhat in advance of number writing.

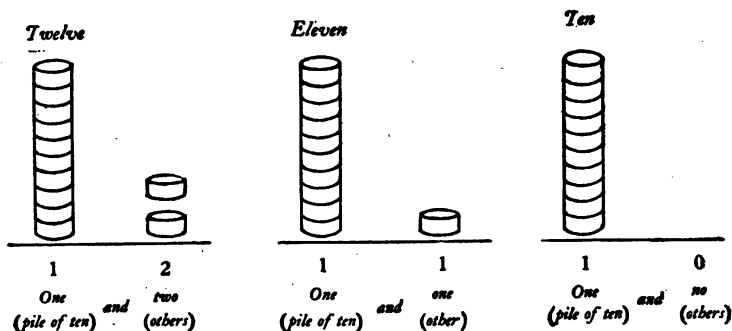
The following points require attention:

*First.* Let all the pupils' number writing be done at the blackboard until nine is reached. The path followed by the pencil point or crayon in forming the figure should be correct from the start; bad habits in this respect can be most easily avoided by having all work done in full view of the teacher.

*Second.* All numbers written by the pupil should represent groups of things counted by him.

When the number exceeds nine, the things, if visible, should be grouped by tens. Thus, if there are twelve objects, ten should be put in the left group and two in the right. If twelve, thirteen, and fourteen are written, (not counted), before ten, the significance of the zero will be more obvious.

### COUNTING OBJECTS AND WRITING NUMBERS ABOVE NINE



### III. SOLUTION OF PROBLEMS BY COUNTING

In all objective solution of problems, it will be found necessary to employ both of the general modes of counting already described. The conditions of the problem will state the degrees of plurality of certain groups or unities, which are to be constructed. The answer will be found by ascertaining the degree of plurality of a group or unity produced by certain manipulations of the objects required by the condi-

tions of the problem. Thus, a pupil is directed to put five pencils in a box. The degree of plurality is given him, and he constructs the group or unity. Another pupil is directed to put four more pencils in the box. The same mode of counting is used. Still another pupil is asked to find how many pencils are now in the box. Mark that the manipulation of pencils by the first two pupils has produced a unity whose degree of plurality the third pupil is required to measure.

Objective solution easily divides itself into three stages, as follows:

*First.* The stage in which real objects such as books, marbles, and pencils should be used, and the movement of these objects called for by the conditions of the problem should actually be performed by the pupils.

*Second.* The stage in which pictures are used.

*Third.* The stage in which counters are substituted for objects, and groups of them representing the unities dealt with are moved together or apart on the number table, so as to suggest the actual movements of the objects called for in the problem.

It will be perceived that the first two stages are merely introductory to the third, and are therefore of short duration. The following points are to be noticed:

(a) The problems are to be announced orally, step by step, i. e., one step is to be performed before the next is announced.



(b) Before passing to the third stage, the pupils should be made somewhat familiar with (1) additive counting, (2) subtractive counting, (3) partitive counting. The following examples will illustrate the character of the exercises:

(1) *Additive counting*

Teacher says, "Mary may move five books from the shelf to the table." Mary does as directed.

"Harry may move two more books from the shelf to the table." Harry does so.

"Grace may find how many books are on the table." Grace counts the books and announces that there are seven.

"Willie may tell us the story." Willie says something equivalent to the following:

"Mary put five books on the table. Harry put two books on the table. Grace counted seven books on the table."

Willie should not be held to any fixed form of expression, nor should his mistakes be made the subject of too much criticism.

(2) *Subtractive counting*

Teacher says, "John has seven marbles in a bag." She gives John a bag and has him put seven marbles into it.

"He gave four of them to George."

George steps forward and receives the four marbles counted out to him by John.

"Now George may find how many marbles John has left in the bag."

George counts and announces that three are left.

"Edna may tell us the story."

Edna says something equivalent to the following:

"John had seven marbles in a bag. He gave four of them to George and has three left in the bag."

### (3) *Partitive counting*

Teacher says, "I have 12 pencils, (counting them in the presence of the class). I wish to give them all to Thomas, Martha, and Edna, so that one has just as many as each of the others. How shall I do it?"

After rejecting several proposed methods of division, teacher says, "I will give one to Thomas, (suiting the action to the word), one to Martha, and one to Edna. Now they all have the same, but I haven't given them all of the pencils. What shall I do next?" After various suggestions, teacher again gives a pencil to each of the three, and after further discussion, repeats the process until the twelve pencils are distributed. "Now, how many have you, Thomas?" Thomas counts and answers, "Four." "How many has Martha, class?" Pupils answer, "Four." "This is a hard story, so I will tell it myself. I had twelve pencils, and wanted to give them to Thomas, Martha, and Edna, so that each would have the same number.

So I gave them one apiece; then another apiece; and so on until all the pencils were gone. We then found that each of them had four pencils."

## 2. SECOND STAGE OF SOLUTION OF PROBLEMS BY COUNTING

In this stage, pupils count pictures of objects instead of the objects themselves. These pictures are drawn upon the blackboard by the teacher as the problems are announced. Thus,

### (1) *Additive counting*

Teacher says, "Here are two fish globes."  
Draws them on the board.



"How many fish in the first globe?"

Draws five, which pupils count.

"How many in the second globe?"

Draws four, which pupils count.

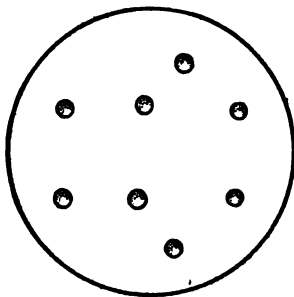
"How many in both globes?"

Pupils count, and answer, "Nine."

(2) *Subtractive counting*

Teacher says, "Two boys together place 8 marbles in a ring."

Draws ring and marbles.



"One of the boys shot 3 marbles out of the ring."

Erases 3 marbles.

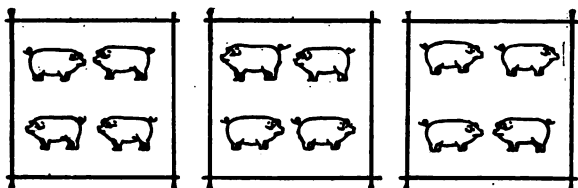
"How many marbles were left in the ring?"

Pupils count, and answer, "Five."

(3) *Partitive counting*

Teacher says, "A farmer has 3 pig pens."

Draws three rectangles.



"He has 12 pigs to put in them."

Draws one pig in each pen.

"How many has he put in now?"

"Three."

Draws one pig more in each.

"How many has he put in now?"

Continue this until there are 12 pigs in the three pens.

"How many in this pen?" (Pointing.)

"Four."

"How many in this?" (Pointing.)

"Four."

"How many in this?" (Pointing.)

"Four."

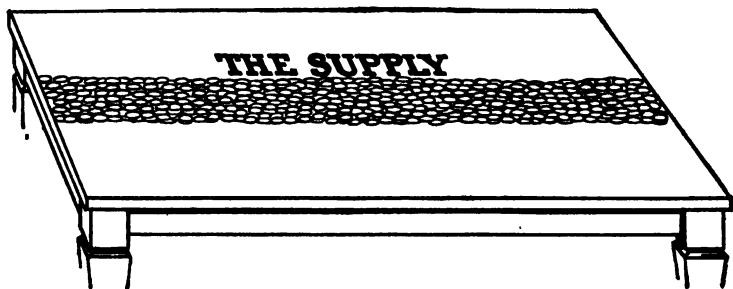
### 3. THIRD STAGE OF SOLUTION OF PROBLEMS BY COUNTING

At this stage the number table and the counter are brought into use. The counter is the universal symbol of any concrete unit. To the child, it may stand for a pencil, a horse, a bird, a book, or any individual object whatever. Later, when artificial pluralities are dealt with, it may represent a pint, a pound, an hour, or even a foot, though linear units are best represented by drawing lines on the blackboard.

The selection of a suitable counter is, therefore, a matter of some importance. An ordinary gun wad will be found very satisfactory. Its size, shape, and noiselessness all recommend it. Any counter may, of

course, be used without seriously affecting the character of the work, but in little things, as in larger ones, there is always room for choice.

### THE NUMBER TABLE AND COUNTERS



In this stage the pupil is taught not only to solve problems by counting but also to write the stories numerically before telling them orally. *To have the mode of counting suggest the mode of writing is the end sought.*

The following examples will illustrate the exercises of this stage:

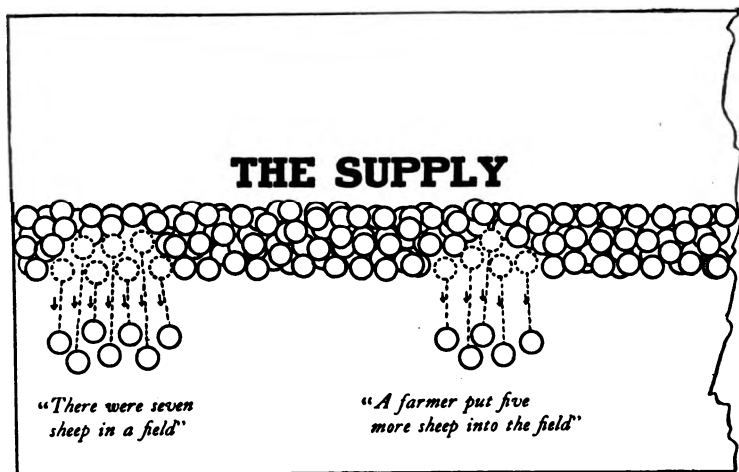
#### (1) *Additive counting and its numerical expression*

The children are standing around the number table, along the middle of which are strewn a sufficient number of counters for their use. These counters should be known and referred to as "the supply." Teacher says, "There were seven sheep in a field. We have no sheep here, so we will pretend each of

these counters is a sheep. You may each take seven from the supply. The pupils do as directed. Teacher, after inspecting the work of each, says, "A farmer put five more sheep into the field. Take five from the supply and put them in the field." The pupils do so.

"Now how many sheep are in the field?"

### ADDITIVE COUNTING (WITH COUNTERS)



*"Now how many sheep are in the field?"*

NOTES. 1. The dotted circles show the places in the supply from which the pupil has moved the counters needed to solve the problem.

2. The dotted lines and the arrows show the direction of each movement.

3. The circles at which the dotted lines terminate show the new positions of the counters.

Pupils count, and the one called upon answers, "Twelve."

"Let us see whether we can write this story on the blackboard. How many sheep were in the field at first?"

Pupils answer, "Seven," and teacher writes 7 on the board.

"How many sheep did the farmer put into the field?"

Pupils answer, "Five," and teacher writes 5 below 7, thus, 7

$$\begin{array}{r} 7 \\ 5 \\ \hline \end{array}$$

"How many sheep did you find were then in the field?"

Pupils answer, "Twelve," and teacher writes 12 below the line, thus, 7

$$\begin{array}{r} 7 \\ 5 \\ \hline 12 \end{array}$$

"When you worked this story, where did you get the seven sheep?"

Pupils answer, "From the supply."

"And where did you get the five sheep?"

Pupils answer, "From the supply."

Teacher says, "Yes, both from the supply," and while so saying, she writes plus before the 5, thus, 7

$$\begin{array}{r} 7 \\ +5 \\ \hline 12 \end{array}$$

"Now listen while I tell the story.



"There were seven sheep in a field, (pointing to the 7 on the board). A farmer put five more sheep into the field, (pointing to the 5). Then there were twelve sheep in the field, (pointing to the 12).

"Mary, will you tell the story?"

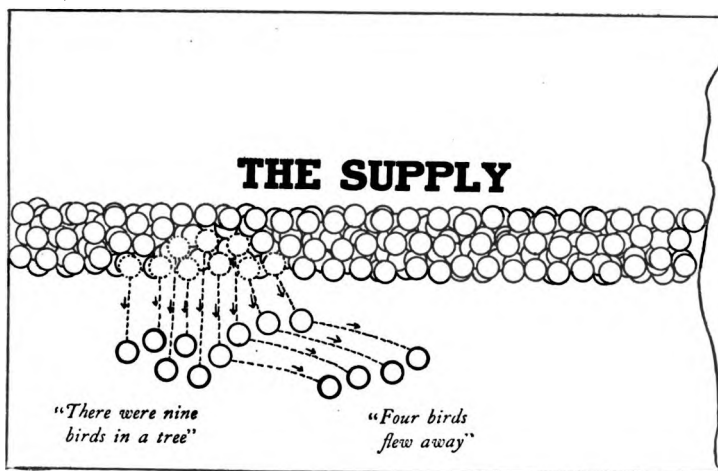
Mary comes forward, takes the pointer, and tells the story, after the pattern set by the teacher.

(2) *Subtractive counting and its numerical expression*

Teacher says, "There were nine birds in a tree. Take them from the supply."

Pupils do as directed.

**SUBTRACTIVE COUNTING**  
(WITH COUNTERS)



"How many birds were left in the tree?"

NOTES. 1. The dotted circles show the places in the supply from which the pupil has moved the counters needed to solve the problem.

2. The dotted lines and the arrows show the direction of each movement.

3. The faint circles show the first stopping places of four of the counters. The heavy circles show the final positions of the counters.

“Four birds flew away. Do you take them from the supply?”

Pupils answer, “No.”

“No; you take them from the nine.”

Pupils do as directed.

“Now find how many birds were left in the tree.”

Pupils count, and answer, “Five.”

“Let us see whether you can write the story.

“How many birds were in the tree at first?”

Pupils answer, “Nine,” and teacher writes 9 on the blackboard.

“How many flew away?”

Pupils answer, “Four,” and teacher writes 4 below 9, thus,

$$\begin{array}{r} 9 \\ 4 \end{array}$$

“How many birds did you find were left in the tree?”

Pupils answer, “Five,” and teacher writes 5 below the line, thus,

$$\begin{array}{r} 9 \\ \hline 4 \\ 5 \end{array}$$

“When you worked this story, where did you get the nine birds?”

Pupils answer, "From the supply."

"Did you take the four birds from the supply?"

Pupils answer, "No."

Teacher says, "No, you took them from the nine birds," and while so saying she writes minus before the 4, thus, 9

$$\begin{array}{r} -4 \\ \hline 5 \end{array}$$

"Now listen while I tell you the story.

"There were nine birds in a tree, (pointing to the 9 on the board). Four of them flew away, (pointing to the 4). So there were five left in the tree, (pointing to the 5).

"John, will you tell the story?"

John comes forward, takes the pointer, and tells the story.

### (3) *Partitive counting and its numerical expression*

Teacher says, "I have twelve apples and wish to put them on three plates, so that one plate will have as many apples on it as each of the others.

"Take the twelve apples from the supply."

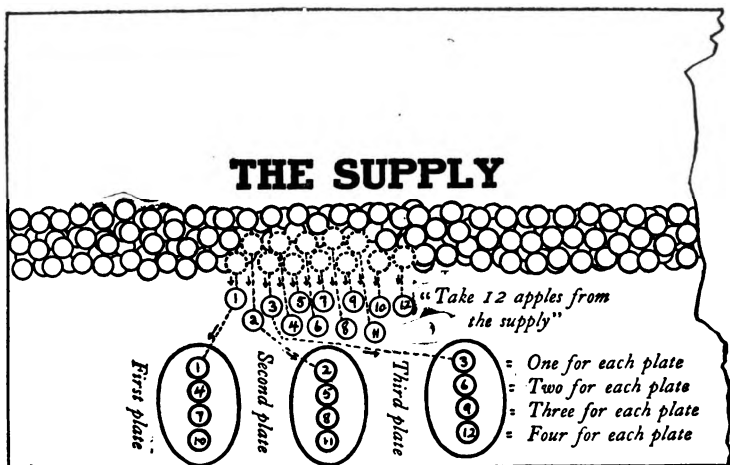
Pupils do so.

"Put one on the first plate, (pupils start first group); one on the next plate, (pupils start second group); and one on the third plate, (pupils start third group)."

"Now put another on the first plate; another on

the next; another on the third." This is continued until the apples are all placed.

### PARTITIVE COUNTING (WITH COUNTERS)



- NOTES.** 1. The dotted circles show the places in the supply from which the pupil has moved the counters needed to solve the problem.
2. The dotted lines and the arrows show the direction of each movement.
3. The faint circles show the first stopping place of each counter. The heavy circles show the final positions of the counters.
4. The order in which the counters are moved from their first stopping places to their final positions on the plates is indicated by their numbers 1, 2, 3 etc., to 12. This mode of counting is called "one at a time into 3 places," and is recorded  $\frac{1}{3}$ .

"Find how many apples are on the first plate."

Pupils count, and answer, "Four."

"How many on the next?"

"Four."

"On the last plate?"

"Four."

"Which plate has the most apples on it?"

"They all have the same."

"Let us see whether we can write the story.

"How many apples did you take from the supply?"

"Twelve."

Teacher writes 12 on the blackboard.

"How many plates did you have to put them on?"

"Three."

Teacher writes 3 on the blackboard, thus,  $\frac{\quad}{3}$

"How many at a time did you put on each plate?"

"One."

Teacher writes 1 over 3, thus,  $\frac{1}{3}$

"When you were through, how many apples were on each plate?"

"Four."

Teacher completes numerical expression, thus,  $\frac{1}{3}$  of 12 = 4.

"Listen while I tell the story. I had twelve apples, (pointing to 12), and wished to put them on three plates, (pointing to 3). So I put one at a time, (pointing to 1), on each plate, (pointing to 3), until the

apples were all gone. Then I found 4, (pointing to 4), on each plate.

“Grace may tell the story.”

NOTE.—It has been objected that this treatment of partitive counting takes no explicit notice of the fact that the major unity twelve has been divided into three equal minor unities, the plurality of each being four. Nor does it take cognizance of the full significance of the symbol  $\frac{1}{3}$ . The first objection is without foundation, as a careful examination of the exercise will reveal. As to the second, the answer is that it would be absurd to attempt such a task. The various meanings of the symbol  $\frac{1}{3}$ , (and there are more of them than most of the objectors know), will be taken up as they are needed in the pupil's development. For the present all that he needs to know is that it records *a mode of counting*.

### Announcement of Problems

It will be noticed that up to this point the problems have been announced step by step, each step being solved before the next is announced. The story has not been told as a whole until after the numerical expression of the solved problem has been written.

Before taking the next step, the teacher should announce problems as wholes and have them repeated by several members of the class before any part of the solution is attempted. Then let each pupil work out the objective solution for himself and write the numerical expression thereof on his slate, or tablet.

When the class can do this well with additive, subtractive, and partitive counting, the next step should be taken.

(4) *The multiplicative expression of a special case of additive counting*

Teacher says, "Edna has two dolls; Martha has three, and Emma has seven. How many dolls have they all?"

"Repeat the story, George."

George does so.

"Class may work it."

Pupils work out the problem with counters, and produce the following on their tablets:

$$\begin{array}{r} 2 \\ 3 \\ +7 \\ \hline 12 \end{array}$$

Teacher says, "I will give you another story.

"Edna has four dolls; Martha has four, and Emma has four. How many dolls have they all?"

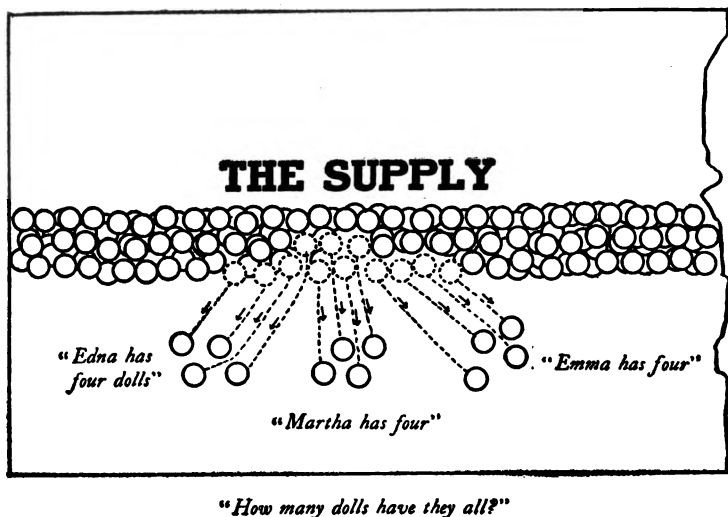
NOTE.—The reader will notice that this problem is just like the preceding one, except that the unities to be added are of the same degree of plurality.

"Repeat the story, Willie."

Willie does so.

"Class may work it."

# ADDITIVE COUNTING—EQUAL PARTS (WITH COUNTERS)



NOTES. 1. The dotted circles show the places in the supply from which the pupil has moved the counters he needed to solve the problem.

2. The dotted lines and the arrows show the direction of each movement.

3. The circles at which the dotted lines terminate show the new positions of the counters.

Pupils work out the problem with counters, and produce the following on their tablets:

$$\begin{array}{r} 4 \\ 4 \\ +4 \\ \hline 12 \end{array}$$



Teacher writes this expression on the blackboard, and then proceeds.

“How many girls are there in this story?”

“Three.”

“How many dolls did each girl have?”

“Four.”

“How many times have I written four on the blackboard?”

“Three times.”

“We may write ‘three times’ this way, (writing on the board,  $3 \times$ ).

“Three times what?”

“Four.”

Teacher writes 4 after the sign of multiplication, making the expression on the board read,  $3 \times 4$ .

“How many dolls did you find they all have?”

“Twelve.”

Teacher completes the expression on the blackboard, making it read,  $3 \times 4 = 12$ .

“Listen while I tell the story. Three girls, (pointing to the 3), had four dolls apiece, (pointing to the 4). So they all had twelve, (pointing to the twelve).”

Thereafter, if the pupil should write the additive expression, show him that since the parts are equal, the short form may be used.

NOTE.—The reader will observe that in order to use the multiplicative expression, the pupil must notice two things not necessarily noticed in using the additive form: (1) that the

parts are equal; (2) how many parts there are. These are the two ideas which at present constitute the pupil's notion of multiplication. It is not necessary at this time that he be given the full arithmetical conception of multiplication, which includes a third idea; namely, that, through the commutative law, the multiplier and the multiplicand may exchange functions. This enlargement of the concept of multiplication makes it something very different from that of addition, but that does not disturb the fact that the two modes of computation have their genesis in additive counting.

(5) *Measurement, the numerical expression of a special case of subtractive counting*

The numerical expression called measurement may be developed from subtractive counting in much the same way as the multiplicative expression was developed from additive counting. Thus:

Teacher announces the following problem:

"I have twelve roses and wish to put them into vases, four in each vase. How many vases will I need?"

"Repeat the story, George."

George does as directed.

"Take the twelve roses from the supply."

Pupils do so.

"How many will you put into the first vase?"

"Four."

"Count them."

Pupils do as directed.

"How many into the next vase?"

"Four."

"Count them."

"How many into the next?"

"Four."

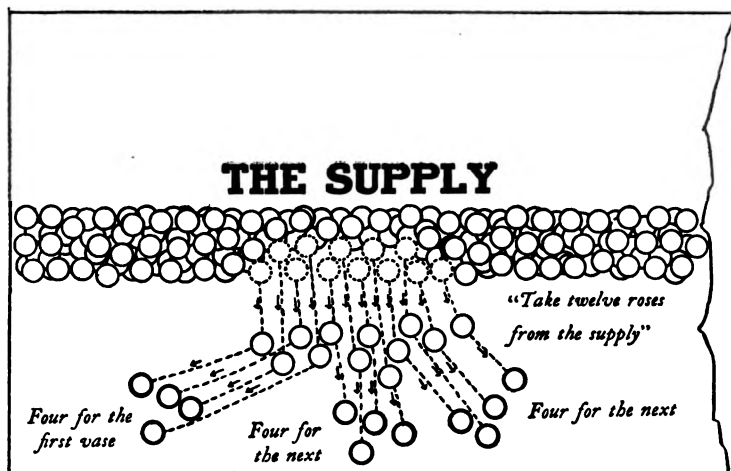
"Count them."

"How many vases have you used?"

"Three."

### SUBTRACTIVE COUNTING

(CONTINUED TAKING AWAY)



"How many vases have you used?"

NOTES. 1. The dotted circles show the places in the supply from which the pupil has moved the counters needed to solve the problem.

2. The dotted lines and the arrows show the direction of each movement.

3. The faint circles show the first stopping places of the counters. The heavy circles show the final positions of the counters.

- “Let us see whether we can write this story.  
“How many roses had we from the supply?”  
“Twelve.”

Teacher writes 12 on the blackboard.

“How many did we take from the 12 for the first vase?”

“Four.”

Teacher writes -4 under the 12, thus,  $\begin{array}{r} 12 \\ -4 \end{array}$

“How many for the next?”

“Four.”

Teacher writes another -4, thus,  $\begin{array}{r} 12 \\ -4 \\ -4 \end{array}$

“How many for the next?”

“Four.”

Teacher again writes,  $\begin{array}{r} 12 \\ -4 \\ -4 \\ -4 \end{array}$

“How many roses were left?”

“None.”

Teacher completes the expression,  $\begin{array}{r} 12 \\ -4 \\ -4 \\ -4 \\ \hline 0 \end{array}$

“How many times did you take away four?”  
(Touching each -4 with the pointer.)

“Three.”

“Then how many vases did you use?”

“Three.”

“I know a better way to write the story. Notice:

“We had twelve roses, (writes 12 on blackboard), and put four into each vase, (writes 4 to left of 12), thus,  $4)12$ . When we had placed all the roses, we found that we had used three vases.” Writes 3 under 12, completing the expression, thus, 
$$\begin{array}{r} 4)12 \\ \underline{\phantom{00}3} \end{array}$$

“Grace may tell the story.”

Grace comes forward, takes the pointer, and tells the story.

### Drills on Numerical Expressions

It is important that the teacher have clearly in mind the vital difference which exists between any two modes of counting and what there is in the numerical expression which indicates this difference, as follows:

(1) Difference between additive and subtractive counting and their expressions.

Suppose the numbers used are nine and seven. In additive counting, both are taken from the supply. In subtractive counting, nine is taken from the supply, and seven from the nine. This difference is shown in the expressions by putting plus before 7 for additive counting and minus before 7 for subtractive counting.

(2) When to use the multiplicative form for additive counting.

When groups taken from the supply are equal, and you count how many groups.

(3) Difference between partitive counting and measurement.

In partitive counting, you put one at a time in several different places.

In measurement, you put more than one at a time in each place.

Exercises like the following will help pupils to notice these differences and will improve the promptness with which the mode of counting suggests the numerical expression.

(1) Teacher writes on the blackboard, 
$$\begin{array}{r} 6 \\ +4 \\ \hline \end{array}$$

"Where did I get the six?"

"From the supply."

"Where did I get the four?"

"From the supply."

Teacher writes on blackboard, 
$$\begin{array}{r} 8 \\ -3 \\ \hline \end{array}$$

"Where did I get the eight?"

"From the supply."

"Where did I get the three?"

"From the eight."

(2) Teacher writes on the blackboard,  $3 \times 4$ .

"Where did I get the four?"

"From the supply."

"How many times did I take four from the supply?"

"Three times."

(3) Teacher writes on the blackboard,  $\frac{1}{4}$

"How many did I count out at a time?"

"One."

"Into how many places?"

"Four."

Teacher writes on the blackboard, 5)

"How many did I count out at a time for each place?"

"Five."

(4) Teacher writes on board, +

Pupils say, "All from the supply."

Teacher writes, —

Pupils say, "Not from the supply."

Teacher writes,  $3 \times$

Pupils say, "Same number from the supply three times."

Teacher writes, 1

Pupils say, "Count one at a time to each place."

Teacher writes, 6)

Pupils say, "Count six at a time to each place."

**Desk Work**

The following is a useful form of desk work. The teacher places on the blackboard such numerical expressions as:

$$\begin{array}{r} 7 \\ +4 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ -3 \\ \hline \end{array}$$

$$\frac{1}{5} \text{ of } 15 =$$

$$5 \times 4 =$$

$$6 \overline{)24}$$

Pupils copy them, work out the answers with counters, and invent stories to fit them.

While this is interesting and profitable busy work, it must be understood that it is merely supplementary to the problem-solving, and not a substitute for that work.

In the latter exercise, the mode of counting suggests the numerical expression. The whole purpose of the work is to enable the pupil to determine which form of numerical expression the problem requires. In the desk work, this central difficulty is overcome for him, the numerical expression being given at the outset.

After five or six months' work upon such exercises as the above, with problems carefully selected and graded, the pupil should be able to describe and perform the mode of counting very promptly, and immediately afterwards to write the numerical expression. When he has attained this degree of proficiency, he is ready for the "Transition Stage of Solution."



## TRANSITION FROM SOLUTION OF PROBLEMS BY COUNTING TO SOLUTION BY COMPUTATION.

This stage begins with the construction and use of the addition and multiplication tables, and ends with the mastery of long division. There is a gradual substitution of imagined counting for actual counting to get the numerical expression, and of computation for counting to reach results.

The pupil stands at the number table with the counters before him, imagines how he would count with them, writes the numerical expression, and consults the addition or the multiplication table for the result.

Theoretically, counters are not used at all, but in practice they will often be employed to refresh the memories of pupils who have partially forgotten the modes of counting.

*First Step.* Make no change in the method of dealing with problems in partition, multiplication, and measurement, but deal with problems in addition and subtraction as described below.

(1) *Addition*

Reserve a place on the board for writing the forty-five combinations.

Know the exact spot where each is to be written. Use any arrangement you prefer. If you have no preference, the following is recommended:

**ADDITION TABLE**

$\frac{1}{+1}$ 2	$\frac{1}{+2}$ 3	$\frac{1}{+3}$ 4	$\frac{1}{+4}$ 5	$\frac{1}{+5}$ 6	$\frac{1}{+6}$ 7	$\frac{1}{+7}$ 8	$\frac{1}{+8}$ 9	$\frac{1}{+9}$ 10
$\frac{2}{+2}$ 4	$\frac{2}{+3}$ 5	$\frac{2}{+4}$ 6	$\frac{2}{+5}$ 7	$\frac{2}{+6}$ 8	$\frac{2}{+7}$ 9	$\frac{2}{+8}$ 10	$\frac{2}{+9}$ 11	
$\frac{3}{+3}$ 6	$\frac{4}{+4}$ 8	$\frac{5}{+5}$ 10	$\frac{6}{+6}$ 12	$\frac{7}{+7}$ 14	$\frac{8}{+8}$ 16	$\frac{9}{+9}$ 18		
$\frac{4}{+3}$ 7	$\frac{5}{+4}$ 9	$\frac{6}{+5}$ 11	$\frac{7}{+6}$ 13	$\frac{8}{+7}$ 15	$\frac{9}{+8}$ 17			
$\frac{9}{+3}$ 12	$\frac{9}{+4}$ 13	$\frac{9}{+5}$ 14	$\frac{9}{+6}$ 15	$\frac{9}{+7}$ 16				
$\frac{8}{+3}$ 11	$\frac{8}{+4}$ 12	$\frac{8}{+5}$ 13	$\frac{8}{+6}$ 14					
$\frac{7}{+3}$ 10	$\frac{7}{+4}$ 11	$\frac{7}{+5}$ 12						
$\frac{5}{+3}$ 8	$\frac{6}{+3}$ 9	$\frac{6}{+4}$ 10						

Suppose the following problem to have been announced: "There are six books on one shelf and five books on another. How many books on both shelves?"

"John, what would you do first?"

"Count six books from the supply."

"Emma, what would you do next?"

"Count five books from the supply."

"Who can write the story that far?"

After hands are shown, a pupil is sent to the board. He writes, 6

$$\begin{array}{r} 6 \\ +5 \\ \hline \end{array}$$

"Edna, what would you do next?"

"Count them together."

"You may all work the story."

Pupils do as directed, and on being questioned, answer that there are eleven books on the two shelves.

Teacher then writes on the blackboard, on the spot reserved for it, the combination, 6

$$\begin{array}{r} 6 \\ +5 \\ \hline 11 \end{array}$$

Afterwards, if a problem should be given requiring the addition of six and five, get the numerical expression as before, by leading pupils to imagine the mode of counting, but instead of counting for the result, find the combination on the board and use the result there recorded.

(2) *Subtraction*

Suppose the following problem to have been announced: "Tom had twelve tops, and sold five. How many tops had he left?"

"Grace, what would you do first?"

"Count twelve tops from the supply."

"George, what would you do next?"

"Count five tops from twelve."

"Who can write the story that far?"

After hands are shown, a pupil is sent to the board. He writes,

$$\begin{array}{r} 12 \\ -5 \\ \hline \end{array}$$

"You may all work and write the story."

Pupils do so, writing,

$$\begin{array}{r} 12 \\ -5 \\ \hline 7 \end{array}$$

**Do Not Record This Expression**

Suppose a problem has been announced which requires the subtraction of five from eleven. Proceed as in the previous problem until you get the expression written on the board,  $\frac{11}{5}$ . Then, instead of telling pupils to work and write the story, proceed as follows:

"Let us see if we can find that from the board."

After some search, the teacher points to the combination, 6

$$\begin{array}{r} +5 \\ \hline 11 \end{array}$$

“Here is eleven, (pointing to 11), and here is five, (pointing to 5). Now, if we took that five from the eleven, what would be left?”

Usually, one or more pupils will answer, “Six.” But if this answer does not come, the teacher must proceed, “I wonder whether it will be six, (pointing to 6). Let us see.”

She then places six counters on the table and five counters near them, and asks, “How many are six and five?”

Pupils answer, “Eleven.”

“I take five from eleven, (removing the five counters). How many are left?”

Pupils answer, “Six.”

“Let us finish writing our story.” Completes the expression, 11

$$\begin{array}{r} -5 \\ \hline 6 \end{array}$$

It will be seen from the above illustrations that the addition table is to be constructed by recording the results of additive counting, not those of subtractive counting. As far as constructed, however, it may be used to avoid the labor of counting both in addition and subtraction. Proceed as above until

about one-third of the addition table is constructed and recorded. By this time pupils will be able to consult it readily both for addition and subtraction. The remainder of the table may then be written in the space reserved for it, and, so far as addition and subtraction are concerned, the pupil is now able to dispense with the use of counters.

*Second Step.* The method of work just described for addition and subtraction is now to be extended to multiplication and both cases of division. The numerical expression is to be determined by imagined counting, and the results of actual counting are to be preserved in the form of the multiplication table. This is to be used, as far as constructed, to avoid the labor of counting. How the table may be so used is explained in Lesson Two, page 19; Lesson Three, page 23; and Lesson Four, page 28, "Numbers Step by Step."

#### IV. COMPUTATION

Computation consists of a series of devices to avoid the labor of counting large numbers.

These devices are known as addition, subtraction, multiplication, and division. It is to be noticed that computation is performed upon *numbers*, counting upon *objects*. Neither additive counting, nor its numerical expression, is addition. That term is

reserved for the mode of computation which takes the place of additive counting. The order of mathematical thought which this book seeks to establish in the minds of primary pupils is as follows: (a) The conditions of the problem suggest a mode of counting; (b) The imagined mode of counting suggests a form of numerical expression; (c) The numerical expression suggests a device of computation. These mental connections are shown in detail in the following table:

MENTAL CONNECTIONS IN PRIMARY NUMBER

Conditions of Problem	Modes of Counting	Numerical Expressions	Modes of Computation
1. Announced step by step	1. Additive Counting	1. Additive Expression	1. Addition
2. Announced as wholes	2. Subtractive Counting	4. Multiplicative Expression	3. Multiplication
		2. Subtractive Expression	2. Subtraction
		5. Measurement	4. Division
	3. Parting	3. Partition	

The results of certain countings have been recorded in the form of the addition and the multiplication tables. The next step is to have pupils memorize the addition table, and then apply it to the addition and subtraction of large numbers.

### 1. Learning the addition table.

The form of addition table already recommended (see p. 38) divided the forty-five combinations into eight groups, as follows:

- (a) Where the smaller addend is 1.
- (b) Where the smaller addend is 2.
- (c) Where the two addends are equal.
- (d) Where the larger addend exceeds the smaller by 1.
- (e) Where the larger addend is 9.
- (f) Where the larger addend is 8.
- (g) Where the larger addend is 7.
- (h) The combinations,

$$\begin{array}{r} 5 \quad 6 \quad 6 \\ +3 \quad +3 \quad +4 \end{array}$$

The table arranged as above is already on the blackboard, and the pupil is consulting it daily in solving concrete problems.

Have the first group memorized and the answers erased.

Before the second group is memorized, the pupils should be exercised in counting by 2's to 20, (2, 4, 6, etc.; also 1, 3, 5, etc.). At first count objects; then without objects.

The third group will be learned with surprising ease.

The fourth group is made easy by associating it with the third; thus,  $3 + 3$  are 6; hence,  $4 + 3$  are 7.

The fifth group should be explained objectively, as follows:



Put on the table ten objects in one group, two in another, and ask pupils to write the number. They will at once write 12, (see p. 11). Now change the grouping so that nine will be in the left group and three in the right. Write on the board,

$$\begin{array}{r} 3 \\ +9 \\ \hline \end{array}$$

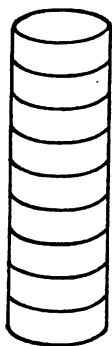
Move one counter from the right group to the left

3

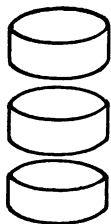
and complete the expression,  $\begin{array}{r} +9 \\ \hline 12 \end{array}$ . After a little ob-

jective practice, the pupils will be able to re-group the numbers in thought without the use of objects.

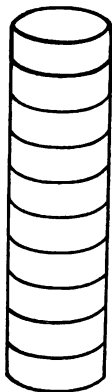
## CHANGE THE 9 TO 10



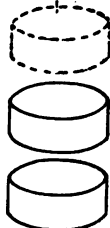
*A pile of 9 and 3 others*



*By moving one of the 3 others to the pile of 9 we have*

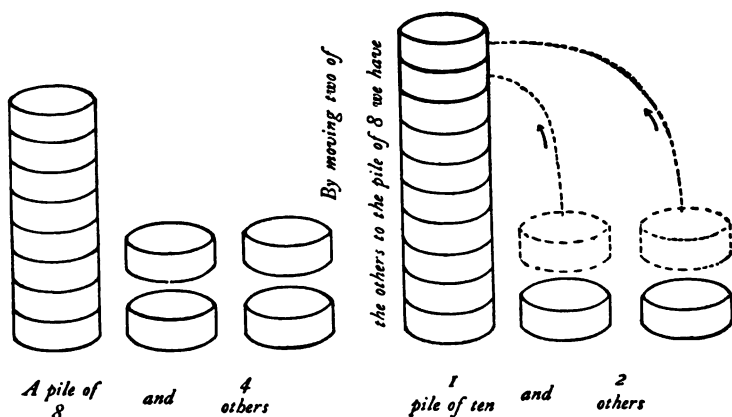


*1 pile of ten and 2 others*



The sixth and seventh groups are taught in the same manner as the fifth; the eighth is taught empirically.

### CHANGE THE 8 TO 10



Other things to be learned and difficulties to be mastered are treated in "Numbers Step by Step."

## V. DIFFICULTIES OF CONCRETE PROBLEMS

1. *Large numbers.*—When concrete problems deal with large pluralities, pupils at first have difficulty in imagining the mode of counting. This difficulty is not to be met by substituting small numbers for the large ones, (a favorite device of those who depend upon verbal arrangement rather than conception of magnitude to determine mathematical relation), but in the manner described below:

Suppose a problem stated which required the addition of 149 and 263.

Teacher says, "What would you do first, Mary?"

"Count 149 from the supply."

Teacher moves (without counting) a large group of counters from the supply, saying, "Well, let us suppose there are 149 counters."

"What will you do next, John?"

"Count 263 from the supply."

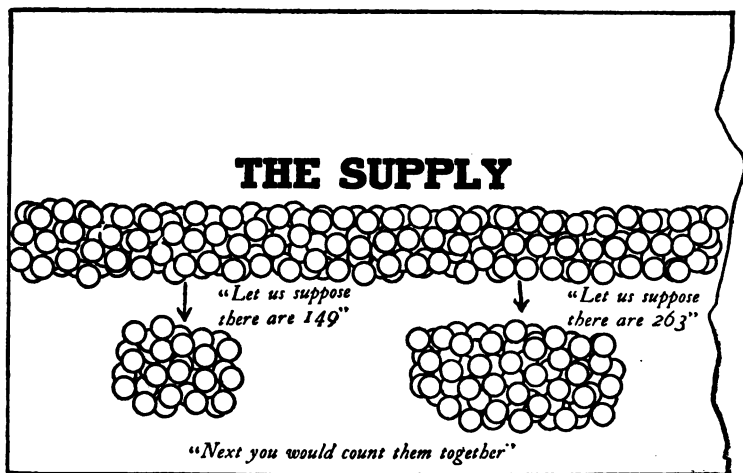
Teacher moves another uncounted group from the supply, and calls it 263.

"What will you do next, Edna?"

"Count them together."

"Who can write this story?"

### LARGE NUMBERS



*"Who can write this story?"*

After hands are shown, a pupil is sent to the black-board. He writes,

$$\begin{array}{r} 149 \\ +263 \\ \hline \end{array}$$

“You may work it.”

Pupils perform the computation of addition upon the numbers written, and announce the result.

In subtraction and multiplication, whenever the use of a large number makes the pupil hesitate in his imagined counting, the teacher should go to the number table, and, by an exercise similar to the one just described, indicate the *mode* of counting by moving together or apart uncounted groups of counters, whose numerical values are assumed to be those employed in the problem.

2. *Comparison of numbers*, (see “Numbers Step by Step,” page 56).

3. *Transformation of function*, (see “Numbers Step by Step,” page 80).

4. *Problems requiring two operations for solution*, (see “Numbers Step by Step,” pages 124, 154, 202).

## VI. DIVISION OF WORK

The ground to be covered in a school year is divided into six parts, each part occupying the time allotted to number work for about six weeks. Since “Numbers Step by Step” is to be put into the hands of pupils at the beginning of the third school year,

this book, "A Primer of Number," undertakes to provide work orally announced for the first two years of school. It will, therefore, be divided into twelve parts.

## PART I

I. Counting to 12. (See pages 6, 7, 8.)

II. Write numbers to 8. (See page 10.)

III. Solve problems by counting real objects at first; afterwards, pictures of objects.

It is to be remembered:

1. That the problems are to be announced orally step by step.

2. That the modes of counting used are:

- (a) Additive. (See page 13.)

- (b) Subtractive. (See page 13.)

3. That no numerical expressions are employed, i. e., pupils are not asked to write problems.

*End.* The end sought is to have each condition of the problem, when announced, suggest an appropriate counting.

## PROBLEMS

### SECOND WEEK

1. *With real objects.*—Henry may take 3 pencils from the box and put them on the table. Mary may take 2 more pencils from the box and put them on

the table. John may count how many pencils are now on the table. Ella may tell the story.

2. *With real objects.*—Sam may put these 4 marbles in one pocket, (handing him the marbles), and these 3 in another pocket. William may count how many marbles Sam has in both pockets. Fred may tell the story.

3. *With real objects.*—Fred may count the marbles in this bag. (Finds 7.) Lucy may take out the glass marbles. (Finds 4.) Mary may count how many are left. John may tell the story.

4. *With real objects.*—Edna may count how many chestnuts are on this plate. (Finds 6.) George may count how many are in this box. (Finds 4.) William may pour the 4 chestnuts from the box to the plate, and see how many are then on the plate. Grace may tell the story.

5. *With real objects.*—Lucy may count the apples on the plate. (Finds 8.) Grace may take 3 from the plate. George may see how many are left. Lucy may tell the story.

### THIRD WEEK

6. *With real objects.*—Henry, and John, and Sam, and William may stand in this aisle. Mary may count how many boys are standing. Fred, and Thomas, and Richard may stand in this aisle. Grace may count them. Alice may count how many boys are standing in both aisles.

7. *With real objects.*—9 boys, (naming them), may stand in the aisle. 3, (naming them), may pass to the blackboard. Mary may count the boys now in the aisle.

8. *With real objects.*—John may count how many chestnuts Fred has in his pocket. (Finds 7.) Fred may give 2 chestnuts to Ella. Sam may find how many chestnuts Fred has left. (Finds 5.)

9. *With real objects.*—Nellie may move 5 red apples from the basket to the plate. Sarah may move 4 green apples from the basket to the plate. Lucy may count how many apples are on the plate.

10. *With real objects.*—Here are 3 boys without pencils. Jane may see how many pencils are in the box. (Finds 10.) Now she may give each boy one. William may find how many pencils are left in the box.

#### FOURTH WEEK

11. *With real objects.*—7 girls, (naming them), may pass into the hall. George may tell 2 of the girls, (naming them), to come back into the room. Sam may find how many girls are now in the hall.

12. *With real objects.*—Lucy, Sarah, and Nellie may go to the blackboard. Henry may count the girls at the blackboard. Grace, Alice, Mary, and Nellie may stand at the number table. George may count the girls at the table. Fred may count all the girls who are standing.

13. *With real objects.*—John may move 6 books from the shelf to the table. Lucy may move 4 more. Ella may count how many books are now on the table.

14. *With pictures.*—Teacher draws on the board three rectangles. In the first she places 4 pigs, in the next 5, and in the third 3. John, how many pigs are in the first pen? Alice, how many are in the second? Mary, how many are in the third? Sam may find how many are in all the pens. Jane may tell the story.

15. *With real objects.*—James may count the books on my table. (Finds 10.) Mary may put 6 of them on the shelf. Grace may count how many books are left on the table. (Finds 4.)

#### FIFTH WEEK

16. *With pictures.*—Here are two rows of soldiers, (pointing to drawing on blackboard). Richard may count the soldiers in the upper row. (Finds 6.) John may count those in the lower row. (Finds 5.) Harry may tell us how many are in both rows. (Finds 11.)

17. *With pictures.*—How many birds (9) are on this telephone wire? 2 of them flew away (erase two). How many are left? (7.)

18. *With pictures.*—Teacher draws two play-houses with 7 dolls in one and 4 dolls in the other. How many dolls in Mary's house? (Pointing to the first.) Class count and pupil called on answers, "Seven." How many dolls in Sarah's house?



(Four.) How many dolls have Mary and Sarah together? (Eleven.)

19. *With pictures.*—Teacher draws two circles with 6 tops in one and 5 in the other. How many tops in Harry's ring? (Pointing to first ring.) How many tops in Tom's ring? How many tops have Harry and Tom together?

20. *With pictures.*—Draw a rectangle with 7 pigs in it. How many pigs in this pen? 4 were sold. (Erase them.) How many are left?

21. *With pictures.*—Draw a house with 9 dolls in it. How many dolls has Ethel? She sold 3. (Erase them.) How many has she left?

22. *With pictures.*—Teacher draws two fish bowls with 4 fish in one and 3 fish in the other. How many fish in Emma's bowl? (Pointing to first.) How many fish in Grace's bowl? (Pointing to second.) How many fish have Grace and Emma together?

### SIXTH WEEK

23. *With pictures.*—Draw a rectangle containing 8 tops. How many tops in this store window? A little boy bought 4 of them. (Erase four.) How many are left?

24. *With pictures.*—Draw 10 soldiers in a row. How many soldiers went into the battle? 4 were wounded. (Erase four.) How many were safe?

25. *With pictures.*—Shades part of the board to represent water, and draws 9 fish. How many fish

in this pond? A boy caught 4. (Erases them.) How many were left in the pond?

26. *With pictures.*—Draw two rectangles with 8 kites in one and 3 in the other. How many kites are flying in this field? (Pointing to the first.) How many kites in this? (Pointing to second.) How many kites are flying in both fields?

27. *With pictures.*—There were 7 leaves on a branch. The wind said, “Come, little leaves,” and 3 went away. How many were left on the branch?

28. *With pictures.*—May may bring me 4 erasers from the front board and 2 erasers from the back board. How many erasers did May bring?

29. *With pictures.*—A little girl made 4 pies in the morning, and in the afternoon she made 3 more. How many did she make altogether?

## PART II

I. Counting to 20.

II. Writing numbers to 15.

III. Solve problems by *using counters*, (see page 17), as well as real objects or pictures.

It is to be remembered:

1. That the problems are to be announced orally, step by step.
2. That the modes of counting used are:
  - (a) Additive.
  - (b) Subtractive.
  - (c) *Partitive*. (See page 14.)

3. That the numerical expressions employed are:

(a) *Additive*. (See page 18.)

(b) *Subtractive*. (See page 21.)

*Ends.* 1. The end sought is to have each condition of the problem, when announced, suggest an appropriate counting.

2. To have each of the first two modes of counting suggest an appropriate numerical expression.

## PROBLEMS

### FIRST WEEK

1. *With real objects*.—6 boys, (naming them), may stand. I wish to send part of them to the board and part to the number table, so that there will be as many at one place as at the other. How shall I do it? (Sends one at a time to each place until all are disposed of.) Mary may count the boys at the blackboard. (Finds 3.) Jane may count the boys at the number table. (Finds 3.) Lucy may tell the story.

2. *With counters*.—There were 8 acorns on the ground. A little chipmunk ran off with 3 of the nuts. How many acorns remained?

3. *With counters*.—Gladys took a basket and went to the hen house and found 8 eggs; on her way home she fell down and broke 2. How many were not broken?

4. *With counters.*—In my new picture book there is a picture of 2 little puppies running after a ball and 5 more puppies asleep on a rug. How many puppies are there in the picture?

5. *With real objects.*—John may count how many marbles are in the box. (Finds 8.) Now he may give them to Harry and William, so that each will have the same number. Harry may tell how many he has. William may tell how many he has. Edna may tell the story.

6. *With real objects.*—George may count the apples in the basket. (Finds 9.) Mary may put them on the 3 plates, so that there will be the same number on each. How many on each plate? (Three.)

7. *With counters.*—Oscar has 3 marbles in his trousers pocket and 4 in his coat pocket. How many marbles has Oscar?

8. *With counters.*—One morning Helen counted 7 robins getting their breakfast in the garden; 3 of them saw the scarecrow and flew away. How many robins were left in the garden?

## SECOND WEEK

9. *With counters.*—May had 4 white aprons, and her mamma made her 3 blue ones. How many aprons had she then?

10. *With real objects.*—Ella may count the chestnuts in this bowl. (Finds 12.) Mary may give them to these 4 children (naming them), so that all will

have the same number. Have each child tell how many he has received.

11. *With counters.*—Some boys had made little sail boats and were sailing them on the pond. They had 8 boats. 2 were upset. How many were left?

12. *With real objects.*—John may find how many pencils are in the box. (Finds 12.) Sam may put them on 3 desks, so that there will be the same number on each desk. Alice may find how many are on each desk.

13. *With counters.*—One morning Helen counted 4 robins in the cherry tree and 3 robins in the strawberry patch. How many robins were in both places?

14. *With counters.*—A mother had 4 girls and 3 boys. How many children did she have?

15. *With real objects.*—Fred may count the books on the desk. (Finds 10.) Ella may put them on 5 shelves, so that there will be the same number on each shelf. Grace may find how many are on each shelf.

16. *With counters.*—Mary went over to the store and bought 9 oranges for her mother. They ate 6 for lunch. How many oranges were left?

### THIRD WEEK

17. *With counters.*—9 girls were jumping rope in front of Helen's house. 3 girls went home. How many girls were left?

18. *With pictures.*—Draw three circles. I have 9 tops and I wish to place them in 3 rings so that

there will be the same number in each. Draw one at a time in each ring until 9 are drawn. How many are in each ring?

19. *With counters.*—On May's birthday cake there were 4 pink candles and 3 green candles. How many candles were on the cake?

20. *With pictures.*—Draw two rectangles. I have 8 pigs and wish to put them into these 2 pens, so that there will be the same number in each, etc.

21. *With pictures.*—By drawing, put 6 birds into 3 cages, putting the same number in each.

22. *With counters.*—There are 4 buttons on Tom's coat, and 5 on Ray's. How many buttons on both boys' coats?

23. *With counters.*—Nellie had 6 handkerchiefs, but she lost 1. How many had she left?

24. *With pictures.*—By drawing, put 12 soldiers into 4 tents, putting the same number in each.

#### FOURTH WEEK

25. *With counters.*—George sawed 4 sticks of wood this morning, and 5 more sticks this afternoon. How many sticks of wood did he saw in all?

26. *With pictures.*—By drawing, put 10 dolls into 5 doll houses, putting the same number in each.

27. *With counters.*—Minnie's geranium had 7 blossoms, but she picked 5 for her auntie. How many blossoms were left?

28. *With pictures.*—By drawing, put 9 fish into 3 fish globes, putting the same number in each.

29. *With counters.*—Mamma has 9 eggs. She wishes to use them all to make 3 cakes, and to put the same number in each cake. How many eggs will there be in each cake?

30. *With counters.*—Tom had 8 marbles, but on his way to school he lost 3 of them. How many marbles had he left?

31. *With counters.*—Helen may give 12 slate pencils to 3 boys—an equal number to each boy. How many pencils will each boy receive?

32. *With counters.*—There were 5 little birds on the ground and 4 sitting on the fence. How many birds were there altogether?

#### FIFTH WEEK

33. *With counters.*—4 girls and 5 boys were playing tag. How many children were in the game?

34. *With counters.*—Tom caught 12 squirrels. He had only 3 cages for them all. How many did he put into each cage, if there were the same number in each?

35. *With counters.*—Dick went fishing with his father. His father caught 8 fish and Dick caught 1. How many did they both catch?

36. *With counters.*—John caught 9 butterflies, but let 5 fly away. How many butterflies did he keep?

37. *With counters.*—A lady had 12 cookies. She gave them to her 3 little boys, so that one had just as many as each of the others. How many did she give each boy?

38. *With counters.*—Susie had 10 paper dolls, but her baby brother spoiled 3. How many paper dolls had she left?

39. *With counters.*—Jennie seated her 12 dolls in 3 chairs, so that there were just as many in one chair as in another. How many dolls were in each chair?

40. *With counters.*—I had 10 walnuts, and cracked 4 of them. How many whole walnuts had I left?

41. *With counters.*—A gardener had 15 rose bushes. He planted the same number in each of 3 flower beds. How many bushes did he put into each bed?

### SIXTH WEEK

42. *With counters.*—Fannie had 12 paper dolls to give 4 girls. She gave the same number to each girl. How many did each one get?

43. *With counters.*—Tom had 7 story books. On his birthday 2 more books were given to him. How many had he then?

44. *With counters.*—Mary picked 4 red roses and 5 pink roses. How many roses did Mary pick?

45. *With counters.*—Mamma ironed 15 napkins and piled them in 3 piles, putting the same number into each pile. How many were in each pile?



46. *With counters.*—Percy had 10 marbles. 4 of them were glass; the others were china. How many of them were china?

47. *With counters.*—A mother cat has 3 white kittens and 3 black kittens. How many kittens has the mother cat?

48. *With counters.*—A little boy had 15 marbles. He put them into his 3 pockets, one at a time, until they were all gone. How many did he put into each pocket, if he put the same number in each?

49. *With counters.*—There were 15 children at Louisa's party, and they ate at 3 tables. How many children ate at each table, if the same number were seated at each table?

50. *With counters.*—George had 10 peanuts. He gave his sister 5 of them. How many had he left?

### PART III

I. Counting to 20.

II. Writing numbers to 20.

III. Solve problems by using counters.

It is to be remembered:

1. That the problems are to be announced *as wholes*.

2. That the modes of counting used are:

(a) Additive.

(b) Subtractive.

(c) Partitive.

3. That the numerical expressions employed are:

- (a) Additive.
- (b) Subtractive.
- (c) *Partitive*. (See page 23.)

The ends sought are:

1. To have each condition of the problem, when announced, suggest an appropriate counting.
2. To have each of the three modes of counting suggest an appropriate numerical expression.

## PROBLEMS

### FIRST WEEK

1. *With counters*.—Mr. Brown had 15 cows. He sold them to 5 men. Each man bought the same number of cows. How many cows did one man buy?

2. *With counters*.—Mary had 5 daisies. She picked 2 more. How many daisies did Mary then have?

3. *With counters*.—Santa Claus had 10 dolls. He gave 5 to some little girls. How many dolls had he left?

4. *With counters*.—There were 6 ladies on the car, and 4 more got on. How many ladies were then on the car?

5. *With counters*.—A lady had 10 spools of thread. She used 4 in making a dress. How many had she left?

6. *With counters*.—There were 12 pieces of cheese

on a plate in the pantry, and the little mice carried away 5 pieces. How many pieces were left?

7. *With counters.*—Grace picked 15 pansies. She gave them to Mary, and Lottie, and Helen, so that each had the same number. How many did she give each?

8. *With counters.*—Harry had 15 watermelon seeds to plant in his garden. He planted them in 3 equal rows. How many seeds were in each row?

9. *With counters.*—I put 6 books on a shelf and 4 books on another shelf. How many books did I put on both shelves?

10. *With counters.*—Willie had 15 firecrackers. He divided them equally among 3 boys. How many did each boy get?

## SECOND WEEK

11. *With counters.*—4 boys and 6 girls went to the park. How many children went to the park?

12. *With counters.*—I have 12 pencils. I am going to give them to Elsie, Edith, and Ruth, so that each child may receive the same number of pencils. How many pencils will each child receive?

13. *With counters.*—There were 13 birds in a cage. Johnny left the cage door open, and 6 birds flew away. How many birds remained in the cage?

14. *With counters.*—6 boys were playing ball; then 5 more boys came to play with them. How many boys were there altogether?

15. *With counters.*—My mamma made 6 mince pies and 5 pumpkin pies for Thanksgiving. How many pies did she make altogether?

16. *With counters.*—Mary picked 15 roses. She gave 9 roses to her mother. How many roses did she have left?

17. *With counters.*—Mamma baked 15 hot cakes, and put them on 3 plates. Each plate held the same number of cakes. How many cakes were on each plate?

18. *With counters.*—Lizzie had 15 chocolate creams; she gave them to her 3 little brothers, giving each the same number. How many did each little boy get?

19. *With counters.*—12 boys were playing “black man.” 4 boys ran through safely. How many were caught?

20. *With counters.*—The postman brought 5 letters in the morning and 6 letters in the afternoon. How many letters did he bring that day?

### THIRD WEEK

21. *With counters.*—Roy found 7 eggs in one nest and 5 in another nest. How many eggs did Roy find?

22. *With counters.*—I had 15 roses. I gave them to 5 little girls, giving the same number to each girl. How many roses did each girl get?

23. *With counters.*—Mamma has 5 bananas on a plate and 7 in a basket. How many bananas has she?

24. *With counters.*—Mamie wiped the dishes for her mother. There were 15 plates. She put them in 3 piles, putting the same number in each pile. How many plates did she put in each pile?

25. *With counters.*—James had 12 chestnuts. He ate 7 of them. How many had he left?

26. *With counters.*—Fred put 12 sticks of wood in his mother's woodbox. She burned 9 sticks while getting dinner. How many sticks were left in the box?

27. *With counters.*—Nellie had 5 blue ribbons and 8 white ones. How many ribbons did she have in all?

28. *With counters.*—There were 5 roses on one bush and 6 roses on another bush. How many roses were on both bushes?

29. *With counters.*—Mary had 12 sticks of candy. She gave 3 to her sister. How many had she left?

30. *With counters.*—Nellie's mamma had 16 doughnuts in the pantry. She told Nellie to put them on 2 plates, so that there would be as many on one plate as on the other. How many doughnuts did Nellie put on each plate?

#### FOURTH WEEK

31. *With counters.*—On a tree there were 14 pears, but the wind blew off 9. How many remained on the tree?

32. *With counters.*—I saw 8 boys playing ball and 3 boys looking on. How many boys did I see?

33. *With counters.*—Harold had 16 pigeons. He

made 4 houses for them, putting just as many into one house as into another. How many were in each house?

34. *With counters.*—A boy raised 12 turkeys, and then sold 8 turkeys. How many turkeys did he have left?

35. *With counters.*—In a flower garden there were 6 roses and 7 pansies. How many flowers in the garden?

36. *With counters.*—8 little girls were playing tag. 6 other little girls came to play with them. How many little girls were then playing tag?

37. *With counters.*—A farmer had 14 cows. His son milked 6. How many were left for the farmer to milk?

38. *With counters.*—There were 12 roses on a bush. I picked 9 of them. How many were left on the bush?

39. *With counters.*—A farmer put 18 eggs into 2 baskets, so that there would be as many in one basket as in the other. How many eggs were in each basket?

40. *With counters.*—Mary had 18 plates; she put them on 3 shelves so that the same number was on each shelf. How many plates were put on each shelf?

#### FIFTH WEEK

41. *With counters.*—Mrs. Hill bought 18 candles for a Christmas tree. They came in 3 boxes with the same number in each. How many candles in a box?

42. *With counters.*—Bertie had 11 picture cards. He gave 5 to Paul. How many cards had he left?

43. *With counters.*—Fred found 7 nuts and Harry found 8 nuts. How many did they both find?

44. *With counters.*—I have 18 pencils to put into 3 boxes. If I put the same number into each box, how many pencils will there be in each box?

45. *With counters.*—My sister had 18 new books. She put them on 3 shelves of her book case, so that the same number was on each shelf. How many books did she put on each shelf?

46. *With counters.*—9 little children were drawing on the board, but the teacher sent 3 of them to their seats. How many were left at the board?

47. *With counters.*—A hen had 11 little yellow chicks. A hawk caught 3. How many chicks were left?

48. *With counters.*—I rode in the street cars 5 times last week and 5 times this week. How many times did I ride in the two weeks?

49. *With counters.*—11 frogs were on a log, and 3 jumped into the water. How many frogs were left on the log?

50. *With counters.*—Tom has 5 white rabbits, 4 gray rabbits, and 3 black rabbits. How many rabbits has he altogether?

## SIXTH WEEK

51. *With counters.*—Peter saw 12 sheep. 7 of them were black and the others were white. How many white sheep did he see?

52. *With counters.*—A lady bought 12 eggs and found that 2 were bad. How many were good?

53. *With counters.*—Aunt Fannie sent 18 bags of candy to 6 children, giving each child the same number. How many bags of candy did she send to each child?

54. *With counters.*—There are 6 chairs in the parlor and 5 in the kitchen. How many chairs in both rooms?

55. *With counters.*—A man had 20 pumpkins. He put them into 4 large boxes, so that each box contained the same number. How many were in each box?

56. *With counters.*—20 children were going for a ride. They climbed into 4 wagons, the same number of children getting into each wagon. How many children were in each wagon?

57. *With counters.*—There were 12 trees in a yard, and a man cut down 4 of them. How many trees were not cut down?

58. *With counters.*—A farmer had 20 cabbages. He put them into 5 baskets, putting the same number in each. How many cabbages did he put in each basket?

59. *With counters.*—When Willie reached school



one morning, he found that 6 boys and 7 girls were already there. How many children were there ahead of him?

60. *With counters.*—I can see from my window 6 maple trees, 3 poplar trees, and 2 elm trees. How many trees can I see from my window?

## PART IV

I. Counting to 40.

II. Writing numbers to 40.

III. Solve problems by using counters.

It is to be remembered:

1. That the problems are to be announced as wholes.
2. That the modes of counting used are:
  - (a) Additive.
  - (b) Subtractive.
  - (c) Partitive.
3. That the numerical expressions called for are:
  - (a) Additive.
  - (b) Subtractive.
  - (c) Partitive.
  - (d) *Multiplicative.* (See page 27.)

The ends sought are:

1. To have each condition of the problem, when announced, suggest an appropriate counting.
2. To have each of the three modes of counting suggest an appropriate numerical expression.

## PROBLEMS

## FIRST WEEK

1. *With counters.*—A lady made 2 bean-bags for each of her 3 little boys. How many bean-bags did she make?

2. *With counters.*—Fred saw 6 men in one car, 4 men in another, and 3 men in another. How many men did Fred see?

3. *With counters.*—David, Joe, and Paul each found 4 ripe strawberries. How many strawberries did they all find?

4. *With counters.*—There are 13 books on a shelf. 5 have red bindings, the others brown. How many books with brown bindings?

5. *With counters.*—I bought 18 stamps at the drug store, and put half of them on some letters. How many did I use?

6. *With counters.*—Edith found 8 pins on the parlor floor and 5 on the dining-room floor. How many pins did she find in all?

7. *With counters.*—There are 7 red stripes and 6 white stripes in our flag. How many stripes in all?

8. *With counters.*—On Willie's birthday his mamma made 15 popcorn balls. He and his play-mates ate 12. How many were left?

9. *With counters.*—Jack had 13 rabbits. 4 died. How many were left?

10. *With counters.*—Della ate 4 chocolate creams, 5 marshmallows, and 3 caramels. How many pieces of candy did she eat?

11. *With counters.*—I have 20 books. If I put them in 4 equal piles, how many books will be in each pile?

12. *With counters.*—There were 3 seesaws in a yard. On each seesaw there were 4 boys. How many boys were playing seesaw?

### SECOND WEEK

13. *With counters.*—A farmer has 6 red calves and 9 red and white ones. How many calves has he in all?

14. *With counters.*—Mary wrote 13 words on the board, and erased 6 of the words. How many words were left?

15. *With counters.*—There are 3 swings in the City Park, and 4 children can swing in each. How many children can swing at the same time?

16. *With counters.*—There were 14 pigs in a pen, and some one let 5 of them out. How many were then in the pen?

17. *With counters.*—Joe, Eddie, and May have each 4 stars. How many stars have they all?

18. *With counters.*—John caught 8 trout, and Willie caught 7 trout. They put them together in one fish basket. How many fish were in the basket?

19. *With counters.*—Little Jack Horner had 3

pies, and took 4 plums out of each pie. How many plums did he take out of all the pies?

20. *With counters.*—Little Bo Peep had 15 sheep. She lost 12 of them. How many did she then have?

21. *With counters.*—A lady had 21 buttons. She put them on 3 aprons for her little girls, putting the same number on each. How many buttons did she put on each apron?

22. *With counters.*—There were 15 lamps in a church; 8 were burning. How many were not lighted?

23. *With counters.*—A farmer had 24 turkeys. He put them into 4 coops, so that there would be the same number in each. How many turkeys did he put into each coop?

24. *With counters.*—Mamma gave Mary 7 hair ribbons, and Grandma gave her 6 hair ribbons. How many ribbons did she have altogether?

### THIRD WEEK

25. *With counters.*—Mamma has 4 plants in each of 3 windows. How many plants has she?

26. *With counters.*—A boy had 20 flags to put in 4 windows. He wished to put the same number in each. How many could he put in each window?

27. *With counters.*—In the morning Herbert counted 16 cherries on his little cherry tree, but while he was at school a robin ate 6 of them. How many were there when he came home?

28. *With counters.*—Willie caught 16 fish. He

gave 7 of them to his grandma for her supper and kept all that were left for his nurse. How many were left for his nurse?

29. *With counters.*—Fred has 9 rabbits in a pen and 4 rabbits in a box. How many rabbits has he altogether?

30. *With counters.*—I have 4 glass globes and 5 goldfish in each globe. How many goldfish have I?

31. *With counters.*—Tom made 8 kites and Willie made 6 kites. How many kites did both boys make?

32. *With counters.*—In a class of 17 children there were 9 boys. How many girls?

33. *With counters.*—Mrs. Brown gave a party. She made 6 apple pies, 5 pumpkin pies, and 4 mince pies. How many pies did she make for her party?

34. *With counters.*—Mamma uses 5 eggs for each cake. How many eggs does she use for 4 cakes?

35. *With counters.*—Clyde's apple has 8 seeds and Cecil's has 7 seeds. How many seeds in both?

36. *With counters.*—The janitor has 20 chairs to put into 4 rows. He wishes to put the same number in each row. How many will be in each row?

#### FOURTH WEEK

37. *With counters.*—There were 8 oranges on a plate, and 3 children each took one. How many oranges were left?

38. *With counters.*—Mrs. Brown made 20 aprons. She gave them to 10 little girls, so that each girl had

the same number. How many aprons did she give one little girl?

39. *With counters.*—A lady went down town with 17 books and came back with 9 books. How many books did she leave down town?

40. *With counters.*—I had 18 nuts, but ate 9 of them and gave the rest away. How many nuts did I give away?

41. *With counters.*—John had 9 rabbits, 4 dogs, and 3 cats. How many pets had he?

42. *With counters.*—Ernest, Harry, and Jack each caught 4 fish. How many did they all catch?

43. *With counters.*—I saw 4 wagons. Each wagon had 4 wheels. How many wheels on the four wagons?

44. *With counters.*—Ella has 9 paper dolls and 7 china dolls. How many dolls has she?

45. *With counters.*—Fred made 24 snowballs. He put them in 3 equal piles. How many snowballs did he put in each pile?

46. *With counters.*—John had 10 problems to work. He worked 6 in the morning. How many problems were left?

47. *With counters.*—Harry caught 9 flies on one piece of flypaper and 7 flies on another. How many flies did Harry catch?

48. *With counters.*—An ant has 6 legs. How many legs have 4 ants?

## FIFTH WEEK

49. *With counters.*—How many ducks do I see, if I see 10 in one yard and 6 in another?

50. *With counters.*—Jennie has 4 dolls. She made 3 dresses for each doll. How many dresses did she make?

51. *With counters.*—There were 12 children in a class. 4 were boys. How many were girls?

52. *With counters.*—Anna divided 24 pieces of candy so that each of her 6 little visitors should have as many as the other. How many pieces did each visitor have?

53. *With Counters.*—Two little boys were playing marbles. One boy put 9 marbles in the ring. The other put in 8 marbles. How many marbles did both boys put in?

54. *With counters.*—A boy misspelled 4 words each day. How many words did he miss in 5 days?

55. *With counters.*—Ned had 15 nuts. He fed 7 of them to his pet squirrel. How many nuts were left?

56. *With counters.*—Mamma cooked 2 potatoes for each child. There were 5 children. How many potatoes did she cook?

57. *With counters.*—Grandma gave Ruth 7 roses, 8 pansies and 4 dahlias. How many flowers did she give her?

58. *With counters.*—Mother put up 15 cans of

fruit. 7 of them were cherries, and the others were peaches. How many were peaches?

59. *With counters.*—5 children were in my room, and I gave one child just as many flowers as I gave the other. How many did each receive if I gave away 25 flowers?

60. *With counters.*—Nellie brought me 6 red roses; Bessie, 5 white roses; and May, 7 pink roses. How many roses did I then have?

#### SIXTH WEEK.

61. *With counters.*—There are 17 chickens in Edith's chicken yard. 9 of the chickens are black and the rest white. How many chickens are white?

62. *With counters.*—A baker made 6 apple pies. In each pie he put 3 apples. How many apples did he use?

63. *With counters.*—I saw 6 ships. Each ship had 3 sails. How many sails on all the ships?

64. *With counters.*—Jane had 17 words to write. If she has written 8, how many has she yet to write?

65. *With counters.*—Grace planted 19 ferns, but only 7 of them lived. How many died?

66. *With counters.*—There were 9 candles on Jack's cake and 11 candles on Eva's cake. How many candles were on both of the cakes?

67. *With counters.*—7 girls each ate the same number of apples. They all ate 21. Each girl ate how many?



68. *With counters.*—There were 6 row boats on the river. 3 men were in each boat. How many men in all of the boats?

69. *With counters.*—There were 12 roses on one bush and 9 roses on another. How many roses on both bushes?

70. *With counters.*—Mrs. Brown had 21 pieces of candy. She gave them all to her 3 children, giving the same number of pieces to each child. How many pieces did each child get?

71. *With counters.*—There were 21 leaves on a bush. 9 of the leaves were brown, and the others were green. How many green leaves were there?

72. *With counters.*—Helen planted 17 ferns, but only 6 of them lived. How many died?

## PART V

I. Counting to 70.

II. Writing numbers to 70.

III. Solve problems by using counters.

It is to be remembered:

1. That the problems are to be announced as wholes.
2. That the modes of counting used are:
  - (a) Additive.
  - (b) Subtractive.
  - (c) Partitive.

3. That the numerical expressions are:

- (a) Additive.
- (b) Subtractive.
- (c) Partitive.
- (d) Multiplicative.
- (e) *Measurement*. (See page 30.)

The ends sought are:

1. To have each condition of the problem, when announced, suggest an appropriate counting.

2. To have each of the three modes of counting suggest an appropriate numerical expression.

## PROBLEMS

### FIRST WEEK

1. *With counters*.—You were given 5 new words on Monday, 6 new words on Tuesday, and 7 new words on Wednesday. How many words in all?

2. *With counters*.—Santa Claus had 21 dolls. He put them on 3 Christmas trees, putting the same number on each. How many dolls on each tree?

3. *With counters*.—Kate can hem 2 towels in one day. How many towels can she hem in 7 days?

4. *With counters*.—James sold 9 newspapers on Monday, 4 on Tuesday, and 8 on Wednesday. How many did he sell in all?

5. *With counters*.—A lady divided 8 cakes among her children. She gave 2 cakes to each child. How many children did she have?

6. *With counters.*—There are 3 eggs in each of 7 nests. How many eggs in all the nests?

7. *With counters.*—A farmer had 21 colts, but sold 8 of them. How many has he left?

8. *With counters.*—Alice had 9 doll dresses. She gave 3 to each doll. How many dolls had she?

9. *With counters.*—A newsboy started out with 24 papers. He sold 20 of them. How many papers were not sold?

10. *With counters.*—Mollie gave each of her little friends 5 chocolate creams. It took 10 pieces. How many of her friends were there?

11. *With counters.*—I gave 2 apples apiece to 5 little girls. How many apples did they all have?

12. *With counters.*—Mamma has 24 peaches. She divided them equally among 3 jars. How many peaches did she put into each jar?

### SECOND WEEK

13. *With counters.*—A house has 9 rooms. 5 are in the first story, and the others in the second story. How many rooms are in the second story?

14. *With counters.*—4 little girls came to my house, and I gave them each 3 apples. How many apples did I give to all?

15. *With counters.*—Mary has 10 dolls. She can put only 2 dolls in a bed. How many beds will she need?

16. *With counters.*—A lady has 12 curtain rings.

She wishes to put them on poles, putting 4 on each pole. How many poles must she have?

17. *With counters.*—Helen had 24 handkerchiefs. She put them into three boxes, putting just as many into one box as into another. How many handkerchiefs did she put into each box?

18. *With counters.*—Mamma put 4 hats in each box. She had 12 hats to put away. How many boxes did she need?

19. *With counters.*—A newsboy started out with 24 papers. He sold 21 of them. How many papers did he take home?

20. *With counters.*—11 chickens were roosting on a pole. A fox caught 2 of them. How many were left?

21. *With counters.*—Fred had 24 blocks. He piled them up one day into 2 equal piles. How many did he put in each pile?

22. *With counters.*—There were 5 girls at a party and each little girl ate 3 dishes of ice cream. How many dishes of ice cream did the girls eat?

23. *With counters.*—Mary had 7 skeins of black wool and 6 skeins of orange wool. How many skeins did she have?

24. *With counters.*—A lady had 3 little girls. Each little girl had 5 dolls. How many dolls had they all?

## THIRD WEEK

25. *With counters.*—Jane had 9 pennies in her bank, and her mother put in 4 more. How many pennies were then in the bank?

26. *With counters.*—Willie divided 24 lettuce leaves equally among his 4 rabbits. How many leaves did each receive?

27. *With counters.*—A watch dealer had 13 watches. 7 of them were gold and the others silver. How many were silver?

28. *With counters.*—There are 5 children on each of 3 benches. How many children on all the benches?

29. *With counters.*—John has 12 hazelnuts for his pet squirrels. He will give 6 nuts to each squirrel. How many squirrels has he?

30. *With counters.*—John had 4 blue flags and 5 red ones. How many flags had he?

31. *With counters.*—Mamma had 12 chairs, and she put 3 chairs in each room. How many rooms did she put them in?

32. *With counters.*—Little Ned had 12 building blocks, but 4 of them fell in the fire and were burned. How many had he then?

33. *With counters.*—I divided 25 pieces of candy equally among 5 children. How many pieces did each one get?

34. *With counters.*—A squirrel has 14 nuts hid-

den in a hole. If he eats 7, how many will be left in the hole?

35. *With counters.*—Mamma sewed 5 roses on each of her 3 little girls' hats. How many roses did mamma sew on the hats?

36. *With counters.*—A farmer has 3 horses in the field and 4 in the stable. How many horses has he?

#### FOURTH WEEK

37. *With counters.*—Charlie's mother baked 11 cakes, and Charlie ate 2 of them. How many were left?

38. *With counters.*—A man gave Fred, Frank and Willie each 5 flags. How many flags did they all have?

39. *With counters.*—A cook used 6 eggs for a pudding and 4 for a cake. How many eggs did she use?

40. *With counters.*—Jane has 27 flowers and is going to put them into 3 vases, putting the same number into each vase. How many flowers in one vase?

41. *With counters.*—There were 4 birds in each of 4 cages. How many birds in all the cages?

42. *With counters.*—How many sticks does it take to make one square? How many squares can you make with 12 sticks?

43. *With counters.*—A lady colored 27 eggs for Easter. She gave them to her 3 little boys, giving

each the same number. How many eggs did each little boy have?

44. *With counters.*—A man bought 12 oranges and ate 4 of them. How many had he left?

45. *With counters.*—I saw 9 carriages in a parade. There were 2 horses to each carriage. How many horses were there?

46. *With counters.*—A gardner planted 28 pansies in 4 rows, putting the same number of pansies in each row. How many pansies in each row?

47. *With counters.*—Henry found 3 eggs on Thursday and 6 on Friday. How many eggs did he find?

#### FIFTH WEEK

48. *With counters.*—There were 6 rooms in the first story of a house, and 4 in the second story. How many rooms in the house?

49. *With counters.*—A farmer put 7 barrels of potatoes into his cellar, and had 4 more to put in. How many barrels of potatoes had he?

50. *With counters.*—Take 15 colored sticks. Put 5 sticks on each slate. Upon how many slates did you put the sticks?

51. *With counters.*—A man planted 3 rows of trees. He put 6 trees in each row. How many trees did he plant in all the rows?

52. *With counters.*—A boy had 16 firecrackers, and shot off 7 of them. How many had he left?

53. *With counters.*—A carpenter made 28 legs for 7 tables. How many was that for each table?

54. *With counters.*—Each of 7 boys erased the same number of words from the board. They stopped when they had erased 28 words altogether. How many words did each boy erase?

55. *With counters.*—James has 15 words to write in rows. He wishes to have 5 in each row. How many rows will there be?

56. *With counters.*—There were 12 collars in a box. 6 of them were taken out. How many remained in the box?

57. *With counters.*—A man has an orchard of 30 trees. The trees are in 6 equal rows. How many trees in each row?

58. *With counters.*—Jessie has 3 shoe strings. She put 6 beads on each string. How many beads on all the strings?

59. *With counters.*—A woman gave each of her little children 5 cookies when they came home from school. How many little children did she have, if she gave away 15 cookies?

60. *With counters.*—A lady made 3 coats for her little boys, and put 6 brass buttons on each coat. How many brass buttons did she use?

#### SIXTH WEEK

61. *With counters.*—If you have 8 large cards and 7 little ones, how many cards have you?



62. *With counters.*—A florist put 30 violets into 5 boxes. He put an equal number of them into each of the 5 boxes. How many were in each box?

63. *With counters.*—A man had 15 apple trees. He planted 5 trees in every row. How many rows did he plant?

64. *With counters.*—A boy had 15 pigeons and sold 7. How many had he left?

65. *With counters.*—There are 8 children in each row of seats. How many children in 4 rows?

66. *With counters.*—32 newspapers were made into 4 packages of equal size. How many papers were in each package?

67. *With counters.*—I gave away 15 pencils, giving 3 to each girl. To how many girls did I give them?

68. *With counters.*—Fred had 15 marbles. He put 5 marbles in each of his pockets. How many pockets had he?

69. *With counters.*—Frank had 12 plums. He gave 5 to his sister and ate the others. How many plums did he eat?

70. *With counters.*—William had 9 white chickens and 8 speckled ones. How many chickens had he?

71. *With counters.*—4 little girls went up on the hill to gather strawberries. Each little girl found 5 berries. How many berries did they all find?

## PART VI

- I. Counting to 100.
  - II. Writing numbers to 100.
  - III. Solve problems by counting.
- It is to be remembered:
1. That the problems are to be announced as wholes.
  2. That the modes of counting used are:
    - (a) Additive.
    - (b) Subtractive.
    - (c) Partitive.
  3. That the numerical expressions employed are:
    - (a) Additive.
    - (b) Subtractive.
    - (c) Partitive.
    - (d) Multiplicative.
    - (e) Measurement.
  4. In addition and subtraction the counting is *imagined*, (see page 37), to obtain the numerical expression, but performed to obtain the result. In partition, multiplication, and measurement the counting is performed as heretofore, to obtain both the numerical expression and the result.

The ends sought are:

1. To have each condition of the problem, when announced, suggest an appropriate counting.

2. To have each of the three modes of counting suggest an appropriate numerical expression.

## PROBLEMS

### FIRST WEEK

1. *With counters.*—A watch dealer had 9 gold watches and 7 silver ones. How many watches had he?

2. *With counters.*—A girl invited 13 friends to a picnic, but 4 of them did not come. How many came?

3. *With counters.*—Carl went to the woodshed 6 times, and brought in 4 sticks of wood each time. How many sticks did he bring in?

4. *With counters.*—There were 20 soldiers standing in 5 equal lines. How many in a line?

5. *With counters.*—There were 8 chickens roosting on one pole and 6 on another. How many were roosting on both poles?

6. *With counters.*—A little girl found 3 hens' nests. There were 7 eggs in each nest. How many eggs did she find?

7. *With counters.*—A little girl put 12 birds into cages. She put 4 birds into each of her cages. How many cages did she have?

8. *With counters.*—Mrs. White took home a bag of peanuts. She had enough to give 7 peanuts to

each of 3 children. How many peanuts were in the bag?

9. *With counters.*—There were 17 telephone poles on a street. A storm blew down 8 of them. How many were left standing?

10. *With counters.*—One train had 9 cars and another 6. How many cars in both trains?

11. *With counters.*—A lady put 22 lilies into 2 vases, so that there were as many in one vase as in the other. How many were in each vase?

12. *With counters.*—16 robins made nests in an orchard. 2 robins live in each nest. How many nests?

### SECOND WEEK

13. *With counters.*—3 little boys went fishing, each little boy caught 7 fish. How many fish did they all catch?

14. *With counters.*—From a street car having 16 passengers, 9 got off. How many remained on the car?

15. *With counters.*—24 nuts were divided equally among 4 children. How many did each receive?

16. *With counters.*—There are 9 girls and 5 boys in a class. How many children are in the class?

17. *With counters.*—I bought 4 bunches of onions from the gardener. There were 6 onions in each bunch. How many onions did I have?

18. *With counters.*—There were 15 pupils in a

class and 6 were at the blackboard. How many were at their desks?

19. *With counters.*—A gardener has 16 trees. He wishes to plant them in rows, planting 8 in each row. How many rows can he make?

20. *With counters.*—John shot his gun at a target 17 times. He hit the target 8 times. How many times did he miss?

21. *With counters.*—A lady hung 16 pictures in her house. She hung 4 pictures in each room. How many rooms in her house?

22. *With counters.*—Susie has 18 apples to put on plates. She wishes to have 6 apples on each plate. How many plates must she have?

23. *With counters.*—5 boys caught 20 fish, each boy catching the same number. How many fish did each boy catch?

24. *With counters.*—There are 9 apples on one limb and 3 on another. How many apples on both limbs?

### THIRD WEEK

25. *With counters.*—3 books are on my table and 8 on a shelf. How many in all?

26. *With counters.*—A farmer had 27 chickens. He put them into 3 boxes. How many in each box, if there was the same number in each?

27. *With counters.*—There were 8 rosebushes in Tom's yard. He picked 3 roses from each bush. How many roses did he pick?

28. *With counters.*—A little girl planted 20 beans. She planted 4 beans in each window box. In how many boxes did she plant beans?

29. *With counters.*—In the morning there were 12 ships in a harbor. At sunset only 3 remained. How many had sailed during the day?

30. *With counters.*—There were 4 large cages at the park. In each cage were 6 bears. How many bears were there in all?

31. *With counters.*—Mr. Smith owned 16 building lots, and sold 7. How many had he left?

32. *With counters.*—We spell 5 new words each day. How many words do we spell in 5 days?

33. *With counters.*—My mamma put 21 pretty candles on the Christmas trees. She put 7 candles on each of the Christmas trees. How many Christmas trees did she have?

34. *With counters.*—I want to put 20 blue beads on some strings, putting 5 on each string. How many strings will I need?

35. *With counters.*—Mary had 3 pencils at school and 7 more at home. How many pencils has she in all?

36. *With counters.*—In a dining room there were 3 tables with 8 persons eating at each table. How many persons were in the room eating?

## FOURTH WEEK

37. *With counters.*—If there are 8 houses on one side of a street and 4 on the other, how many houses are on that street?

38. *With counters.*—Jane has 27 flowers, and is going to put them into 3 vases, putting the same number into each vase. How many flowers in one vase?

39. *With counters.*—A lady bought 12 fruit jars. She broke 4 of them, and put up fruit in the others. How many jars of fruit did she put up?

40. *With counters.*—A janitor had 20 desks. He wishes to put them into 4 equal rows. How many will there be in each row?

41. *With counters.*—Ella has 8 blue buttons and 5 glass ones. How many buttons has she?

42. *With counters.*—An engine was taking 4 flat cars down the track. On each car there were 7 men. How many men were on all the cars?

43. *With counters.*—If a baby should drink 3 cups of milk every day, how many days would it take the baby to drink 21 cups?

44. *With counters.*—Charlie took 5 marbles out of a bag, and 7 were left in the bag. How many marbles had he?

45. *With counters.*—Our coffee-pot holds 8 cups of coffee. Papa drank 2 cups. How many cups were left?

46. *With counters.*—Hanna's mamma put 30 crackers on plates, placing 5 on each plate. How many plates did she have?

47. *With counters.*—A farmer had 13 sheep and sold 7. How many had he left?

48. *With counters.*—There were 9 plates at a table. If it took 27 spoons to set the table, how many spoons were placed at each plate, if the same number were placed at each plate?

#### FIFTH WEEK

49. *With counters.*—A lady put 24 glasses of jelly on 4 shelves, so that there would be the same number on each shelf. How many glasses were on each shelf?

50. *With counters.*—A girl put 4 rows of beads, with 7 beads in each row, on her doll's cap. How many beads on the cap?

51. *With counters.*—A lady has 7 new collars and 4 old ones. How many collars has she?

52. *With counters.*—A lady bought 24 cookies and told the man to put 8 cookies in each bag. How many bags did he use?

53. *With counters.*—Minnie saw 10 stars, but soon a cloud covered all of them but 3. How many did the cloud cover?

54. *With counters.*—Molly bought 25 oranges. She will put 5 oranges on each plate. How many plates will she need?



55. *With counters.*—A little boy planted 28 squash seeds in 7 hills, using the same number of seeds for each hill. How many seeds did he put into a hill?

56. *With counters.*—One of our cats caught 6 mice, and the other caught 5. How many mice did both cats catch?

57. *With counters.*—Frank had 15 apples. If he ate 3 a day, in how many days will he eat them all?

58. *With counters.*—Harold has 9 buttons on each coat. How many buttons has he on 3 coats?

59. *With counters.*—Will planted 17 violet plants, and 8 of them died. How many lived?

60. *With counters.*—5 boys each sold 6 papers. How many papers did they all sell?

### SIXTH WEEK

61. *With counters.*—If 20 people go to a picnic on the river, and each row boat holds 4 persons, how many boats will they have to rent?

62. *With counters.*—In my house there are 6 rooms, with 5 chairs in each room. How many chairs have I?

63. *With counters.*—On one branch of a rose bush there were 9 buds, on another branch there were 2 buds. How many buds were on both branches?

64. *With counters.*—A newsboy bought 17 papers in the morning, and he had 8 left at noon. How many papers did he sell?

65. *With counters.*—Charles had 3 goldfish in one glass and 7 in another. How many had he in both glasses?

66. *With counters.*—Mr. Smith had 5 sons and 4 daughters. How many children had he?

67. *With counters.*—A gardener put 30 radishes into 5 equal bunches. How many were in each bunch?

68. *With counters.*—A merchant had 7 barrels of flour and bought 9 more. How many barrels had he then?

69. *With counters.*—John has 7 white rabbits and 4 gray ones. How many rabbits has he?

70. *With counters.*—Harry was given 10 words to spell. He spelled them all but 3. How many did he spell?

71. *With counters.*—Blanche had 35 chrysanthemums. She put them into 5 vases, the same number in each. How many in each vase?

72. *With counters.*—There were 6 vases on the table. Each vase had 5 roses in it. How many roses in all the vases?

## PART VII

I. and II. Reading and writing numbers to 200.

III. Solve problems by counting.

Notice:

1. That the problems are to be announced as wholes.

2. That the modes of counting used are:
    - (a) Additive.
    - (b) Subtractive.
    - (c) Partitive.
  3. That the numerical expressions employed are:
    - (a) Additive.
    - (b) Subtractive.
    - (c) Partitive.
    - (d) Multiplicative.
    - (e) Measurement.
  4. In all operations the numerical expressions are obtained by imagined counting.
- In addition and subtraction the table on page 38 is put on the board and the results obtained from it.
- In partition, multiplication, and measurement the results are still obtained by counting.
- IV. Groups (a), (b), and (c) of addition table are memorized.

## PROBLEMS

### FIRST WEEK

1. *Table*.—Mary has 11 dolls. They are all paper dolls but 4. How many paper dolls has she?
2. *With counters*.—Edith eats 3 meals a day. How many meals will she eat in 5 days?
3. *Table*.—A man keeps 2 horses to drive and 8 to work. How many horses has he?

4. *With counters.*—I have 6 Indian baskets. In each basket there are 5 shells. How many shells altogether?

5. *With counters.*—A baker had 30 eggs, and he put 6 eggs into each cake. How many cakes did he bake?

6. *With counters.*—A man gave 4 flags to each of 5 boys. How many flags did he give?

7. *With counters.*—A farmer took 4 loads of wood to town one week; 5 loads the next week; and 3 loads the next. How many loads did he take to town altogether?

8. *With counters.*—A man put 45 melons into 9 baskets, putting the same number into each basket. How many did he put in each basket?

9. *Table.*—When Mary started to school she counted 15 cherries on her cherry tree. When she came home only 6 were left. How many of her cherries were taken while she was at school?

10. *With counters.*—Santa Claus had 50 tin soldiers. He gave them to 5 boys, the same number to each boy. How many did each boy receive?

11. *With counters.*—There are 2 boys at the blackboard, 5 at the number table, and 6 at their desks. How many boys in all?

12. *With counters.*—A boy pulled up 20 turnips. He tied them in bunches, 5 in a bunch. How many bunches did he have?

13. *With counters.*—In a school room there are

30 desks. 3 of them are not used. How many are used by pupils?

14. *Table*.—Willie found 6 chestnuts and 5 walnuts. How many nuts did he find?

15. *Table*.—George had 11 books. 8 of them were new, and the others old. How many of his books were old?

## SECOND WEEK

16. *Table*.—A lady bought a box containing 12 collars. Now there are only 4 left in it. How many has she taken out?

17. *With counters*.—3 boys made 5 snowballs apiece. How many snowballs in all?

18. *Table*.—7 girls in a class were promoted and 6 failed. How many girls were in the class?

19. *With counters*.—A man hitched 12 horses to 3 wagons, putting the same number in each team. How many horses were in a team?

20. *Table*.—A boy put 5 cents in his money box. There were already 7 cents in it. How many cents are now in it?

21. *With counters*.—Henry has 24 wheels which he is going to use to make wagons. He is going to make four-wheeled wagons. How many wagons will he have?

22. *With counters*.—In a dining room there were 3 tables, with 6 persons eating at each table. How many persons were in the room eating?

23. *Table*.—A farmer hired 14 men, but 5 of them left because they found the work too hard. How many remained at work?

24. *Table*.—How many candles are there on a Christmas tree, if 9 candles are lighted and 7 are not lighted?

25. *With counters*.—There were 36 chairs at 9 tables, the same number at each. How many chairs at each table?

26. *Table*.—Ella had 15 car tickets on Monday and only 7 on Thursday. How many tickets had been used?

27. *With counters*.—A cook had 24 oranges, and used 8 oranges each day for baking cake fillings. How many days did the oranges last?

28. *Table*.—Mark had 8 pieces of candy and ate 3. How many pieces of candy had he left?

29. *With counters*.—Sam counted 7 birds on one telephone wire; 4 on another; and 6 on another. How many birds did he count in all?

30. *With counters*.—How many panes of glass are needed for 3 windows, if each window is to have 8 panes?

### THIRD WEEK

31. *Table*.—William gathered 11 eggs in the barn. He dropped 2 of them on the way from the barn to the house. How many whole eggs did he bring to the house?

32. *With counters.*—How many trees are there in 4 rows, if there are 8 trees in each row?

33. *With counters.*—15 people went into a dining room, but  $\frac{1}{3}$  of them had to wait. How many had to wait?

34. *Table.*—A woman had 11 turkeys. She sold 9 of them. How many turkeys were left?

35. *Table.*—Henry saw 5 boys riding bicycles and 7 boys walking. How many boys did he see?

36. *Table.*—A merchant had 7 barrels of flour and bought 9 more. How many barrels had he then?

37. *With counters.*—If the grocer puts 3 melons in one basket, how many baskets will he need for 27 melons?

38. *With counters.*—A farmer has 4 hens, and each hen has 8 little chickens. How many little chickens has he?

39. *With counters.*—The grocer brought us 24 apples yesterday. We have used 9 of them. How many are left?

40. *Table.*—There were 6 pears in one basket and 9 in another. How many in both baskets?

41. *With counters.*—In an orchard were 80 trees. There were 8 rows with the same number in each. How many trees in a row?

42. *Table.*—There were 20 chairs in a room, and 10 were carried out. How many chairs were left?

43. *With counters.*—There were 30 snails in a field, and some ducks ate the snails for breakfast.

Each duck ate 5 snails. How many ducks were there?

44. *With counters.*—Charles read 5 books one week, 3 the next, and 4 the next. How many books did he read in the three weeks?

45. *With counters.*—If there are 7 words on each line, how many words are on 5 lines?

#### FOURTH WEEK

46. *With counters.*—There are 4 rosebushes in front of our house, 5 on one side, and 6 on another. How many rosebushes are in our yard?

47. *Table.*—There are 13 pupils in our Sunday school class. 8 of them are girls. How many are boys?

48. *With counters.*—I set out 5 rows of pansies, putting 7 pansies in each row. How many pansies did I set out?

49. *Table.*—A girl had 8 pieces of candy. She gave her little sister 2 pieces. How many pieces of candy had she left?

50. *With counters.*—30 little girls went for a car ride, 6 little girls sat on each seat. How many seats did they need?

51. *With counters.*—Last week John worked 12 number stories. This week he has worked 14. How many number stories has he worked in both weeks?

52. *With counters.*—A wood sawyer sawed 9



sticks of wood into 27 equal pieces. How many pieces was that for each stick?

53. *With counters.*—When Alice's dress was new there were 36 buttons on it. 17 buttons were lost. How many were then on the dress?

54. *With counters.*—Mrs. Green made bread 5 times last week, and each time she made 6 loaves. How many loaves did she make?

55. *Table.*—There were 9 panes in one window and 4 in another. How many panes in both windows?

56. *With counters.*—Jennie gathered 32 roses. She took them to the hospital and gave 8 roses to each little sick girl until all were gone. To how many little sick girls did she give roses?

57. *Table.*—14 boys were playing ball in a field. 6 had to go home. How many were left in the game?

58. *With counters.*—There are 5 shelves in a room with 7 books on each shelf. How many books are there?

59. *With counters.*—A lady put 32 jars of peaches on 4 shelves, the same number on each shelf. How many jars did she put on each shelf?

60. *Table.*—John had 8 old books, and on Christmas he was given 3 new ones. How many books had he then?

## FIFTH WEEK

61. *With counters.*—Mabel wrote 8 words on her slate, each word having five letters in it. How many letters in all the words?

62. *With counters.*—A boy had 18 pet squirrels. He put them into 3 cages, placing the same number in each. How many squirrels were in each cage?

63. *Table.*—Jennie has 6 glass buttons. Bertha has 9. How many glass buttons have both girls?

64. *Table.*—There were 12 ripe cherries on Jacob's little cherry tree. The birds ate 8. How many were left?

65. *With counters.*—There were 35 children in a schoolroom. If 7 are seated in each row, how many rows are there?

66. *With counters.*—A postman left 3 letters at one house, 6 at the next, and 4 at the next. How many letters did he leave at the three houses?

67. *With counters.*—There were 4 rows of desks in a schoolroom, with 10 desks in each row. How many desks were in the room?

68. *Table.*—Mary has 16 birds. 7 of them can sing. How many of her birds cannot sing?

69. *With counters.*—There were 20 pupils in a class, and  $\frac{1}{4}$  of them were absent. How many were absent?

70. *With counters.*—I picked 5 red leaves from each of 9 trees. How many red leaves did I pick?

71. *Table*.—Annie saw 13 roses on a bush. She picked 5 of them for her mother. How many roses were left on the bush?

72. *Table*.—Sam and Tom were playing ball. Sam hit the ball 6 times, and Tom hit it 8 times. How many times did they both hit it?

73. *Table*.—Fifteen birds were flying across the street. 9 of them alighted on a telephone wire. How many kept on flying?

74. *With counters*.—Ruby wrote 36 lines by writing 9 lines a day. How many days did it take to write them?

75. *Table*.—Martin used 6 kite sticks one day and 9 the next. How many kite sticks did he use on both days?

### SIXTH WEEK

76. *With counters*.—One day Harry went to the store for his mother 3 times. Each time he went he bought 12 doughnuts. How many doughnuts did he buy altogether?

77. *With counters*.—If I divide 36 crackers equally among 5 boys and 4 girls, how many crackers will each child get?

78. *Table*.—A farmer had 8 cows in one lot and 7 cows in another lot. How many cows did he have in both lots?

79. *With counters*.—Elmer had 36 marbles. He asked his mother to make him enough bags so that

he could put 12 marbles in each bag. How many bags did it take?

80. *Table*.—I have 9 red roses and 7 white ones in a vase. How many roses in the vase?

81. *With counters*.—A teacher gave away 48 cards, giving the same number to each of 6 boys. How many cards did each boy receive?

82. *Table*.—James had 7 marbles in one pocket and 5 marbles in another pocket. How many marbles did he have in both pockets?

83. *With counters*.—Henry sawed 8 sticks of wood on Monday, 9 sticks on Tuesday, and 4 sticks on Wednesday. How many sticks of wood did he saw on the three days?

84. *With counters*.—A florist had 40 Easter lilies and sold 37. How many had he left?

85. *With counters*.—A blacksmith used 40 horse-shoes. He put 4 shoes on each horse. How many horses did he shoe?

86. *With counters*.—There were 50 people in a street car. 11 left the car at Main Street. How many remained in the car?

87. *With counters*.—Jane wrote 12 words each day. How many did she write in 4 days?

88. *With counters*.—Grandpa has 48 turkeys to sell. He wants to put them in boxes so there will be 8 turkeys in each box. How many boxes will he need?

89. *Table*.—I had 15 pansies in my garden. The

snails ate 6 of them. How many pansies were left in my garden?

90. *With counters.*—There were 8 benches in the park. 6 persons sat on each bench. How many persons were sitting on all the benches?

## PART VIII

I and II. Reading and writing numbers to 300.  
III. Solve problems by counting.

It is to be remembered:

1. That the problems are to be announced as wholes.
  2. That the modes of counting used are:
    - (a) Additive.
    - (b) Subtractive.
    - (c) Partitive.
  3. That the numerical expressions employed are:
    - (a) Additive.
    - (b) Subtractive.
    - (c) Partitive.
    - (d) Multiplicative.
    - (e) Measurement.
  4. In all operations, the numerical expressions are obtained by imagined counting.
- In addition and subtraction, the results are obtained from the table on the black-

board, except in such combinations as have been memorized.

In partition, multiplication, and measurement, results are still obtained by counting.

IV. Review (a), (b), (c) of addition table and learn (d), (e), (f).

## PROBLEMS

### FIRST WEEK

1. *With counters.*—A girl put 20 oranges into 2 baskets. She put the same number into each basket. How many oranges did she put into each basket?

2. *With counters.*—John gave Frank 10 marbles and Fred 7. How many marbles did he give to both?

3. *With counters.*—If there are 6 handkerchiefs in each box, how many are there in 7 boxes?

4. *With counters.*—John has 4 cents in his pocket and 28 cents in his bank. There are how many times as many cents in his bank as in his pocket?

5. *With counters.*—George had 27 picture cards. He gave 8 to his little brother. How many had he left?

6. *With counters.*—A farmer planted 5 rows of apple trees. In each row there were 12 trees. How many trees did he plant?

7. *With counters.*—Ada had 6 apples, Mary had

8 apples, and Nellie had 10 apples. How many apples did they all have?

8. *With counters.*—A boy gave away 56 pieces of candy. He gave 8 pieces to each of his little friends. How many friends were there?

9. *With counters.*—William planted 42 apple seeds in 7 equal rows. How many seeds in each row?

10. *With counters.*—A boy had 29 chestnuts. He ate 12 of them. How many had he left?

11. *With counters.*—8 boys went fishing. Each boy caught 9 fish. How many fish did they all catch?

12. *With counters.*—Mr. Hall had a boat in which he carried people across a stream. He could only take 9 across at a time. How many trips would he make to take 54 people across?

13. *With counters.*—In going to school, Harry rides 11 blocks and walks 3. How many blocks is it from his home to the school?

14. *With counters.*—A woman gave 18 walnuts to her 2 little girls. She gave the same number to each. How many walnuts did each girl receive?

15. *With counters.*—There are 40 apples in a box. 32 are sweet and the rest are sour. How many are sour?

## SECOND WEEK

16. *With counters.*—A girl owned 6 little books. Each one had 12 pages in it. How many pages in all?

17. *Table.*—9 bars of soap were left in one box

and 6 bars in another. How many bars were left in both boxes?

18. *With counters.*—A street car has 8 wheels. How many cars will 40 wheels supply?

19. *Table.*—I have 17 slate pencils. 8 of them have not been sharpened. How many have been sharpened?

20. *With counters.*—George has 21 marbles and William has  $\frac{1}{3}$  as many. How many marbles has William?

21. *With counters.*—Mother gave 4 apples to Mary, 3 apples to James, 5 apples to Willie, and 6 apples to Anna. How many apples did she give to all the children?

22. *With counters.*—Each of 4 boys made 6 snowballs. How many snowballs did they all make?

23. *Table.*—On a bush there were 12 roses. Mary cut off 8 roses. How many roses did she leave on the bush?

24. *Table.*—There were 5 quarts of milk in one bucket and 3 quarts in another. How many quarts of milk were in both buckets?

25. *With counters.*—There are 48 trees in 6 equal rows. How many trees are there in each row?

26. *With counters.*—There are 4 lions in each of 3 cages. How many lions in all of the cages?

27. *Table.*—Lucy had 16 pieces of candy. She gave her playmates 9 pieces. How many pieces did she keep for herself?



28. *With counters.*—In one house there are 24 windows. In another house there are 8 windows. There are how many times as many windows in the first house as in the second?

29. *With counters.*—Into how many bags must I put 72 cookies, if I put 9 in each bag?

30. *With counters.*—John put 40 lead pencils into boxes. He put 8 pencils in each box. How many boxes did he use?

### THIRD WEEK

31. *With counters.*—There are 9 panes of glass in each window. How many panes in 6 windows?

32. *With counters.*—A man put 12 legs on some tables. He put 4 legs on each table. How many tables were there?

33. *With counters.*—Willie has 16 white marbles and 19 brown ones. How many marbles has he?

34. *With counters.*—A tailor had 36 buttons to sew on 4 coats. He put the same number on each. How many did he sew on each coat?

35. *Table.*—Samuel had 17 marbles in a bag. 8 of them were lost through a hole. How many had he then?

36. *With counters.*—At a game of ball there are 9 boys on each side. How many boys in the game?

37. *With counters.*—Florence put 20 books into 5 desks. She put the same number into each desk. How many books did she put into each desk?

38. *With counters.*—One milk can holds 7 gallons of milk. Another can holds 5 gallons, and another 6 gallons. How many gallons do all the cans together hold?

39. *With counters.*—A groceryman sold 18 bushels of potatoes to some men. He sold 3 bushels to each man. How many men were there?

40. *Table.*—Our cat had 7 kittens. We gave away 6 of them. How many kittens were left?

41. *With counters.*—If there are 10 handkerchiefs in each box, how many are in 9 boxes?

42. *Table.*—There are 16 pupils in our class. Yesterday 9 of them were dismissed early. How many had to stay?

43. *With counters.*— $\frac{1}{3}$  of the books in our library belong to me. How many books have I, if there are 30 books in our library?

44. *With counters.*—There are 19 birds in a tree, 10 on a fence, and 1 on the ground. How many birds altogether?

45. *With counters.*—Harry put 21 rabbits into some boxes. He put 7 rabbits into each box. How many boxes did he use?

#### FOURTH WEEK

46. *With counters.*—Tom has 4 times as many marbles as Harry. Harry has 8. How many has Tom?

47. *With counters.*—Helen had 19 beads on a string. She lost 6 of them. How many had she left?

48. *With counters.*—Paul had 32 marbles which he divided equally among 4 little friends. How many marbles did each one receive?

49. *With counters.*—In a bouquet there were 9 pink roses, 2 red roses, and 7 yellow roses. How many roses were in the bouquet?

50. *With counters.*—Mary had 36 books. She put 9 on each shelf. How many shelves did she use?

51. *With counters.*—Our baseball team made 18 runs, each boy making 2 runs. How many boys in our team?

52. *With counters.*—Three boys went fishing. The first boy caught 12 fish, the second caught 5, and the third caught 20. How many did they all catch?

53. *With counters.*—A gardener planted 27 seeds in 3 flower beds. He put the same number of seeds into each bed. How many seeds were planted in each bed?

54. *With counters.*—There were 25 children in a class. 12 of them were at the number table. The others were at their desks. How many were at their desks?

55. *With counters.*—Helmet has 2 pockets in his jacket. He has 8 marbles in each pocket. How many marbles has he?

56. *With counters.*—20 little candles were burning

on George's Christmas tree. 7 candles went out. How many were still burning?

57. *With counters.*—Mrs. Smith canned 8 pints of strawberries in 4 jars of the same size. How many pints were in each jar?

58. *With counters.*—A hunter shot 16 ducks one day, 20 another day, and 12 another day. How many ducks did he shoot in the three days?

59. *With counters.*—A father, mother, and 3 baby squirrels lived in a hole. Each one had 3 grains of corn for dinner. How many grains of corn did the entire family have for dinner?

60. *With counters.*—In a class there are 7 boys and 14 girls. There are how many times as many girls as boys?

#### FIFTH WEEK

61. *With counters.*—A man bought three sacks of potatoes. One sack held 8 pecks, one held 6 pecks, and one 4 pecks. How many pecks of potatoes in all three of the sacks?

62. *With counters.*—Henry had 5 coops of chickens. In each coop there were 7 chickens. How many chickens did he have in all the coops?

63. *With counters.*—Dora picked 28 quarts of raspberries in 4 days. She picked the same number of quarts each day. How many quarts a day did she pick?

64. *Table.*—Hazel cut out 17 paper dolls. She

gave 8 of them to May. How many dolls did she have left?

65. *With counters.*—Mrs. Brown uses 12 eggs for 4 cakes. How many eggs does she use for each cake, if she uses the same number for each?

66. *With counters.*—A workman had 28 panes of glass to put into the windows of a cottage. He put 4 panes in each window. How many windows were there?

67. *With counters.*—One day a baker baked 25 loaves of white bread and 16 loaves of brown bread. How many loaves of bread did he bake?

68. *With counters.*—A girl used 3 sheets of paper for each letter. How many sheets were required for 9 letters?

69. *With counters.*—35 soldiers slept in 7 tents, the same number sleeping in each tent. How many soldiers slept in each tent?

70. *With counters.*—In a cloakroom are 40 hooks. 25 of them are used by the girls. How many hooks are used by the boys?

71. *With counters.*—William and Harry went out to shoot with a rifle. William hit the mark 3 times and Harry 12 times. Harry hit it how many times as often as William?

72. *With counters.*—Mary strung 25 beads. Carrie strung 15 beads. Jennie strung 19 beads. How many did they all string?

73. *With counters.*—Yesterday Fred and his little

brother together carried 25 sticks of wood into the house. Fred carried 20 sticks. How many sticks did his little brother carry?

74. *With counters.*—A groceryman had 4 cans of coffee. There were 6 pounds of coffee in each can. How many pounds of coffee had he altogether?

75. *With counters.*—Mary's new book has 48 chapters in it. Mary is reading 4 chapters a day. How many days will it take her to finish the book?

### SIXTH WEEK

76. *Table.*—We bought 10 tons of coal. 3 tons were hard coal and the rest soft coal. How many tons of soft coal did we buy?

77. *With counters.*—Dora can make 5 handkerchiefs each week. How many can she make in 3 weeks?

78. *Table.*—There were 9 pounds of tea in one can and 8 pounds in another. How many pounds in both cans?

79. *With counters.*—At a party there were 24 children. The lady wished to seat 4 children at each table. How many tables would she need?

80. *With counters.*—A woman picked 40 quarts of berries in five days. She picked the same number of quarts each day. How many quarts a day did she pick?

81. *With counters.*—A dressmaker used 8 spools

of thread each week. How many spools did she use in 4 weeks?

82. *Table*.—Harry has 6 red stars, 1 green star, 5 blue, and 8 gold ones. How many stars has he in all?

83. *With counters*.—How many trips would a farmer have to make to bring 42 bushels of potatoes to town, if he brings 7 bushels each trip?

84. *Table*. A lady used 12 pounds of sugar in putting up fruit. 5 pounds were white and the rest brown. How many pounds of brown sugar did she use?

85. *With counters*.—A baker sold 42 lemon pies to 7 stores. How many was that for each store, if all the stores bought the same number?

86. *Table*.—Mary has 12 books. 3 of them are new and the others old. How many old books has she?

87. *With counters*.—A woman gave away 4 apples to each of 6 boys. How many apples did she give away?

88. *Table*.—8 boys were flying kites in one field and 6 in another. How many boys in both fields?

89. *With counters*.—Samuel had 14 apples. He ate 2 apples each day till they were all gone. How many days did his apples last?

90. *With counters*.—There were 84 pieces of chalk in a box. It lasted 7 weeks. If the same number of pieces were used each week, how many pieces of chalk were used in one week?

## PART IX

I and II. Reading and writing numbers to 400.

III. Solve problems by counting.

It is to be remembered:

1. That the problems are to be announced as wholes.
2. That the modes of counting used are:
  - (a) Additive.
  - (b) Subtractive.
  - (c) Partitive.
3. That the numerical expressions employed are:
  - (a) Additive.
  - (b) Subtractive.
  - (c) Partitive.
  - (d) Multiplicative.
  - (e) Measurement.

4. In all operations, the numerical expressions are obtained from imagined counting.

In addition and subtraction, the results are obtained from the table on the black-board, except in such combinations as have been memorized.

In partition, multiplication, and measurement, results are still obtained by counting.

IV. Review (a), (b), (c), (d), (e), and (f) of addition table, and learn (g) and (h).



## PROBLEMS

## FIRST WEEK

1. *Table*.—Walter has 12 rabbits. 9 of them are young rabbits and the others are old. How many old rabbits has he?

2. *With counters*.—108 eggs were packed in 9 boxes of an equal size. How many eggs were in each box?

3. *With counters*.—3 bushels fill a barrel. How many bushels will it take to fill 12 barrels?

4. *With counters*.—Some boys were spinning tops. In one ring there were 7 tops; in another ring, 6 tops; in another, 4 tops. How many tops were in the three rings?

5. *With counters*.—The postman brought 9 letters in the morning, 6 at noon, and 7 in the afternoon. How many did he bring altogether?

6. *With counters*.—A teacher gave 18 stars. She gave 3 stars apiece to some little girls. How many girls received the stars?

7. *With counters*.—Maud had 9 handkerchiefs in each of 4 boxes. How many had she altogether?

8. *Table*.—A farmer had 9 cows, but sold 2 of them. How many had he left?

9. *With counters*.—A farmer brought 60 turkeys to market. 12 were in each box. How many boxes of turkeys had he?

10. *With counters.*—A doctor left 36 powders for a sick child to take in 9 days. He had to take the same number of powders every day. How many did he take a day?

11. *With counters.*—A lady had 2 dozen buttons on a card, but used 9 of them. How many had she left?

12. *With counters.*—A milkman served 9 quarts of milk each week to each of 8 families. How many quarts of milk did he serve to them all in a week?

13. *Table.*—Harry had 9 rabbits and 3 dogs. How many pets had he?

14. *With counters.*—A gardener planted 55 rose bushes, putting 11 bushes in each row. How many rows were there?

15. *With counters.*—I wish to divide 56 strawberries equally among 7 children. How many must I give to each child?

## SECOND WEEK

16. *With counters.*—A little boy read 9 books. There were 6 stories in each book. How many stories did he read?

17. *With counters.*—A baker made 19 apple pies, 14 mince pies, and 12 lemon pies. How many pies did he make?

18. *With counters.*—Mary has cut out 36 cookies. She will bake 12 of them in each pan. How many pans does she need?

19. *Table*.—Harry's young cherry tree had 16 ripe cherries on it. Harry picked 9 of them. How many were left on the tree?

20. *With counters*.—Forty-eight boys were rowing in 12 boats. Each boat held the same number of boys. How many boys were in a boat?

21. *With counters*.—A man picked 7 branches from a cherry tree. There were 8 cherries on each branch. How many cherries did he pick?

22. *Table*.—Mother bought 18 peaches and used 9 of them. How many had she left?

23. *With counters*.—At Bessie's birthday party she gave away 45 paper dolls. She gave 5 to each little girl. How many girls were there?

24. *Table*.—On a shelf are 5 red books and 8 brown books. How many books are on the shelf?

25. *With counters*.—On the table were 3 plates full of cakes. Each plate held 7 cakes. How many cakes were on the table?

26. *With counters*.—55 trees were planted in 11 equal rows. How many trees were in a row?

27. *Table*.—George saw 9 sheep in a field. 2 of them were black and the others white. How many of the sheep were white?

28. *With counters*.—A boy pulled 20 radishes. He put them in bunches having 4 radishes in a bunch. How many bunches did he make?

29. *With counters*.—John caught 9 flies on one piece of flypaper, 7 flies on another piece, and 8 flies

on another piece. How many flies did he catch altogether?

30. *With counters.*—A lady divided 24 cookies among 4 girls. All of the girls had the same number of cookies. How many had each?

### THIRD WEEK

31. *With counters.*—There were 36 children in a class. The teacher had them stand in 4 equal rows. How many children were in each row?

32. *With counters.*—Alice pasted 5 cards on each page of her scrap book. How many cards did she paste on 9 pages?

33. *With counters.*—There were 9 candles on Jack's cake, and 13 candles on Mary's cake. How many candles on both cakes?

34. *With counters.*—A man gave away 18 flags. He gave 3 to each little boy he met. How many boys received the flags?

35. *Tables.*—Edna's father gave her a box holding 12 lead pencils. She gave 4 of them to her friends. How many pencils did she keep for herself?

36. *With counters.*—5 boys caught 9 fish apiece. How many fish did they catch altogether?

37. *With counters.*—7 pansies have 35 petals. How many petals is that for each pansy?

38. *With counters.*—A gardener had 27 celery plants. He planted them 9 in a bed. How many beds of celery plants did he have?

39. *With counters.*—There were 20 ounces of candy in one box, 15 ounces in another, and 18 ounces in another. How many ounces of candy in the three boxes?

40. *With counters.*—A baker put 9 loaves of bread into each of 2 baskets. How many loaves did he put into both baskets?

41. *With counters.*—There are 20 sacks of coal in a ton. Bessie counted the sacks the man brought from the wagon into our basement. There were only 18. If we ordered a ton, how many sacks does the coal dealer owe us?

42. *With counters.*—There were 5 red spots on each rubber ball. On all the balls there were 15 red spots. How many balls were there?

43. *With counters.*—28 men went riding in 4 row-boats, with the same number in each boat. How many men were in each boat?

44. *With counters.*—Some boys were having a snowball fight. There were 9 boys on one side and 12 on the other. How many boys were in the fight?

45. *Table.*—Mother bought a dozen eggs and cooked 5 for breakfast. How many had she left?

#### FOURTH WEEK

46. *With counters.*—Mrs. Miller bought 5 cards of buttons. On each card there were 6 buttons. How many buttons did she buy?

47. *Table.*—There were 14 girls in our class, but

6 of them left school. How many girls were then in the class?

48. *With counters.*—A druggist put 24 bottles of medicine into 4 boxes. How many bottles were put into each box, if the boxes all held the same number of bottles?

49. *Table.*—We have 12 fruit trees in our yard. Last year 5 of them did not bear fruit. How many trees had fruit on them?

50. *With counters.*—Some boys went fishing. Each boy caught 6 fish. If they all caught 18 fish, how many boys were there?

51. *With counters.*—In one chicken house there were 11 little chickens; in another house there were 9 little chickens; and in another there were 12 little chickens. How many little chickens were in the three houses?

52. *With counters.*—4 little girls were playing with paper dolls. Each girl had 5 dolls. How many dolls did they have altogether?

53. *With counters.*—Daisy gave 18 shells to 6 girls. How many was that apiece, if each girl received the same number?

54. *Table.*—In Esther's set of doll dishes were 7 pieces. In Mary's set there were 9 pieces. How many pieces in the two sets of dishes?

55. *With counters.*—At Kate's party there were 24 children. 6 of them sat at each little round table. How many tables were there?

56. *With counters.*—A lady cooked 7 eggs each day for 4 days. How many eggs did she cook altogether?

57. *With counters.*—A boy had 20 pigeons in a pigeon house. He sold 12 of them. How many of them were left?

58. *With counters.*—3 boys had altogether 15 tops. If they all had the same number, how many tops had each?

59. *Table.*—Along one side of our yard there are 9 raspberry bushes. On the other side there are 7 raspberry bushes. How many bushes are on both sides of the yard?

60. *With counters.*—Bobby's mother gave him 36 clothespins to play with. It took 9 clothespins to make a pig pen. How many pig pens could he make with all the clothespins?

#### FIFTH WEEK

61. *With counters.*—A lady had 3 little girls. She made each one of them 6 dresses. How many dresses did she make?

62. *With counters.*—In a store window were three jars of candy. One jar held 12 sticks of candy; one held 9 sticks, and one held 8 sticks. How many sticks of candy were in the three jars?

63. *With counters.*—A man built 21 houses, 7 on a block. On how many blocks did he build?

64. *Table.*—A doctor left 15 pills for a sick man.

9 were to be taken in the morning and the others in the afternoon. How many pills did the sick man take in the afternoon?

65. *With counters.*—15 eggs were found in 5 nests. Each nest held the same number of eggs. How many eggs were in each nest?

66. *With counters.*—A lady baked 4 pans of cookies. There were 6 cookies in each pan. How many cookies did she bake?

67. *Table.*—My sister used 14 clothespins to fasten the washing to the clothesline. The wind blew 5 of the clothespins to the ground. How many remained in place?

68. *With counters.*—Mary picked 35 flowers and made them into bunches, 5 flowers in each bunch. How many bunches did she make?

69. *With counters.*—21 cakes were divided among 7 children so that they had the same number apiece. How many cakes had each child?

70. *Table.*—Mary's cat caught 6 mice. Jennie's cat caught 5 mice. How many mice did both cats catch?

71. *With counters.*—A lady gave 4 bananas to each of her 5 boys. How many bananas did she give them all?

72. *Table.*—There were 15 rosebuds on a bush in our yard, but 6 of them did not bloom. How many roses bloomed on the bush?

73. *Table.*—Last month in our room 5 boys and



9 girls had perfect lessons. How many pupils were perfect?

74. *With counters.*—A dressmaker had 54 yards of cloth. She cut it into dress lengths of 9 yards each. How many dress lengths did she cut?

75. *With counters.*—The children were given 3 plums each. It took 12 plums for them all. How many children were there?

### SIXTH WEEK

76. *With counters.*—It took 28 nails to fasten 4 boards. How many nails was that for each board, if each took the same number?

77. *With counters.*—In our room at school there are 6 girls in one row of desks, 7 girls in the next row, and 5 girls in the next. How many girls are in the three rows of desks?

78. *With counters.*—Bessie, Fay, and Mary each had 5 pieces of candy. How many pieces had they altogether.

79. *Table.*—John had 8 Easter eggs. 3 of them were red and the others purple. How many of them were purple?

80. *With counters.*—A lady used 32 eggs making cakes. She put 4 eggs into each cake. How many cakes did she make?

81. *With counters.*—Ethel gave 4 roses apiece to 3 little girls who came to see her. How many roses did she give them all?

82. *With counters.*—Mary brought 6 dolls to a doll party. Susie brought 8 dolls, and Jane brought 4. How many dolls did the three girls bring?

83. *With counters.*—A farmer sold 33 pounds of butter to some grocers, 11 pounds to each one. How many grocers bought the butter?

84. *With counters.*—There are 12 flags on 3 wagons. All the wagons have the same number of flags. How many flags on each wagon?

85. *Table.*—Willie and George played ball. Together they hit the ball 16 times. Willie hit it 7 times. How many times did George hit it?

86. *With counters.*—Sarah picked 2 bunches of daisies with 7 in each bunch. How many daisies did she pick?

87. *Table.*—Martha took 7 cents from her bank. If there were 12 cents in the bank at first, how many were left?

88. *With counters.*—A teacher divided 18 pencils equally among 9 children. How many was that apiece?

89. *With counters.*—A milliner bought 54 violets. They were in bunches of 9 violets each. How many bunches of violets did she buy?

90. *With counters.*—A little girl had a pin store. On one day she took in 7 pins; on another day she took in 5 pins; on still another day she took in 8 pins. How many pins did she take in on the three days?

## PART X

I and II. Read and write numbers to 400.

III. Solve problems as described below.

It is to be remembered:

1. That the problems are to be announced orally as wholes. At this point many teachers begin to prepare for the written or printed announcement of problems by writing on the blackboard the numbers used as the problems are announced. When the pupil is asked to repeat the problem (tell the story again), he has to supply the words which connect these numbers. Care should be taken that no fixed order is followed in writing the numbers. In the addition of two numbers sometimes the larger and sometimes the smaller should be written first. In subtraction sometimes the minuend and sometimes the subtrahend should precede. In multiplication sometimes the multiplier and sometimes the multiplicand. In division sometimes the dividend and sometimes the divisor. The purpose of this variation in order is to exclude all dependable clues to the numerical expression except the imagined mode of counting.

2. That the modes of counting are:
  - (a) Additive.
  - (b) Subtractive.
  - (c) Partitive.
3. That the numerical expressions employed are:
  - (a) Additive.
  - (b) Subtractive.
  - (c) Partitive.
  - (d) Multiplicative.
  - (e) Measurement.
4. That in all operations the numerical expressions are suggested by imagined counting. That in addition and subtraction the results are obtained from the memorized forty-five combinations.

That in multiplication, measurement, and partition results are obtained by actual counting until pupils are taught to use the tables.

- IV. 1. Review the addition table and apply the commutative law, as follows:

The second combination of group (a) should be written  $+\frac{2}{1}$  as well as  $+\frac{1}{2}$ . Pupils should be made to see that these are two ways of writing one combination, not two different combinations. The same thing is of course true of all the thirty-six combinations composed of two different num-

bers. Nine of the forty-five combinations ( $\begin{smallmatrix} 1 \\ +1 \end{smallmatrix}$   $\begin{smallmatrix} 2 \\ +2 \end{smallmatrix}$   $\begin{smallmatrix} 3 \\ +3 \end{smallmatrix}$   $\begin{smallmatrix} 4 \\ +4 \end{smallmatrix}$   $\begin{smallmatrix} 5 \\ +5 \end{smallmatrix}$   $\begin{smallmatrix} 6 \\ +6 \end{smallmatrix}$   $\begin{smallmatrix} 7 \\ +7 \end{smallmatrix}$   $\begin{smallmatrix} 8 \\ +8 \end{smallmatrix}$   $\begin{smallmatrix} 9 \\ +9 \end{smallmatrix}$ ), are constructed by repeating a number, so that what is said of  $\begin{smallmatrix} 1 \\ +2 \end{smallmatrix}$  does not apply to them. While using the addition table to obtain results in addition, the pupil has become somewhat familiar with this fact. Now care should be taken in drill that sometimes one form is used and sometimes the other. Pupils will in this way get a practical knowledge of what mathematicians call the commutative law of addition.

3. Extend groups (a), (b), and (c) as follows:

### GROUP (a)

#### FIRST COMBINATION

1	1	1	1	1	1	1	1	1	1
+1	+11	+21	+31	+41	+51	+61	+71	+81	+91
<hr/> 2	<hr/> 12	<hr/> 22	<hr/> 32	<hr/> 42	<hr/> 52	<hr/> 62	<hr/> 72	<hr/> 82	<hr/> 92

#### SECOND COMBINATION

1	1	1	1	1	1	1	1	1	1
+2	+12	+22	+32	+42	+52	+62	+72	+82	+92
<hr/> 3	<hr/> 13	<hr/> 23	<hr/> 33	<hr/> 43	<hr/> 53	<hr/> 63	<hr/> 73	<hr/> 83	<hr/> 93

#### THIRD COMBINATION

1	1	1	1	1	1	1	1	1	1
+3	+13	+23	+33	+43	+53	+63	+73	+83	+93
<hr/> 4	<hr/> 14	<hr/> 24	<hr/> 34	<hr/> 44	<hr/> 54	<hr/> 64	<hr/> 74	<hr/> 84	<hr/> 94

## FOURTH COMBINATION

1	1	1	1	1	1	1	1	1	1
+4	+14	+24	+34	+44	+54	+64	+74	+84	+94
<u>5</u>	<u>15</u>	<u>25</u>	<u>35</u>	<u>45</u>	<u>55</u>	<u>65</u>	<u>75</u>	<u>85</u>	<u>95</u>

## FIFTH COMBINATION

1	1	1	1	1	1	1	1	1	1
+5	+15	+25	+35	+45	+55	+65	+75	+85	+95
<u>6</u>	<u>16</u>	<u>26</u>	<u>36</u>	<u>46</u>	<u>56</u>	<u>66</u>	<u>76</u>	<u>86</u>	<u>96</u>

## SIXTH COMBINATION

1	1	1	1	1	1	1	1	1	1
+6	+16	+26	+36	+46	+56	+66	+76	+86	+96
<u>7</u>	<u>17</u>	<u>27</u>	<u>37</u>	<u>47</u>	<u>57</u>	<u>67</u>	<u>77</u>	<u>87</u>	<u>97</u>

## SEVENTH COMBINATION

1	1	1	1	1	1	1	1	1	1
+7	+17	+27	+37	+47	+57	+67	+77	+87	+97
<u>8</u>	<u>18</u>	<u>28</u>	<u>38</u>	<u>48</u>	<u>58</u>	<u>68</u>	<u>78</u>	<u>88</u>	<u>98</u>

## EIGHTH COMBINATION

1	1	1	1	1	1	1	1	1	1
+8	+18	+28	+38	+48	+58	+68	+78	+88	+98
<u>9</u>	<u>19</u>	<u>29</u>	<u>39</u>	<u>49</u>	<u>59</u>	<u>69</u>	<u>79</u>	<u>89</u>	<u>99</u>

## NINTH COMBINATION

1	1	1	1	1	1	1	1	1	1
+9	+19	+29	+39	+49	+59	+69	+79	+89	+99
<u>10</u>	<u>20</u>	<u>30</u>	<u>40</u>	<u>50</u>	<u>60</u>	<u>70</u>	<u>80</u>	<u>90</u>	<u>100</u>

Not only does the commutative law change the second combination of group (a) from  $\underline{+1}_2$  to  $\underline{+2}_1$ , but the commuted form of the combination has as many extensions as the original form. Thus besides the extensions give above, we have,

$$\begin{array}{cccccccccc} 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ \hline +11 & +21 & +31 & +41 & +51 & +61 & +71 & +81 & +91 \\ 13 & 23 & 33 & 43 & 53 & 63 & 73 & 83 & 93 \end{array}$$

This is of course true of the extensions of all of the thirty-six combinations composed of two different numbers. In drilling on such extensions this fact should be borne in mind.

It will doubtless occur to the teacher that in extending the forms  $\underline{+1}_2$  and  $\underline{+2}_1$  that the upper addend instead of the lower may be increased by successive additions of ten. Thus, from the form  $\underline{+1}_2$  we may obtain  $\underline{+11}_2$   $\underline{+21}_2$   $\underline{+31}_2$  etc. From the form  $\underline{+2}_1$  we may obtain  $\underline{+12}_1$   $\underline{+22}_1$   $\underline{+32}_1$  etc. But the forms thus derived are not used in practice. In (1) adding a column, and (2) in multiplying when the multiplicand has more than one place, it is always the smaller number to be added to the larger. In our drill, therefore, we need not consider the last two sets of extensions.

## GROUP (b)

## FIRST COMBINATION

2	2	2	2	2	2	2	2	2	2
+2	+12	+22	+32	+42	+52	+62	+72	+82	+92
<hr/> 4	<hr/> 14	<hr/> 24	<hr/> 34	<hr/> 44	<hr/> 54	<hr/> 64	<hr/> 74	<hr/> 84	<hr/> 94

## SECOND COMBINATION

2	2	2	2	2	2	2	2	2	2
+3	+13	+23	+33	+43	+53	+63	+73	+83	+93
<hr/> 5	<hr/> 15	<hr/> 25	<hr/> 35	<hr/> 45	<hr/> 55	<hr/> 65	<hr/> 75	<hr/> 85	<hr/> 95

## THIRD COMBINATION

2	2	2	2	2	2	2	2	2	2
+4	+14	+24	+34	+44	+54	+64	+74	+84	+94
<hr/> 6	<hr/> 16	<hr/> 26	<hr/> 36	<hr/> 46	<hr/> 56	<hr/> 66	<hr/> 76	<hr/> 86	<hr/> 96

## FOURTH COMBINATION

2	2	2	2	2	2	2	2	2	2
+5	+15	+25	+35	+45	+55	+65	+75	+85	+95
<hr/> 7	<hr/> 17	<hr/> 27	<hr/> 37	<hr/> 47	<hr/> 57	<hr/> 67	<hr/> 77	<hr/> 87	<hr/> 97

## FIFTH COMBINATION

2	2	2	2	2	2	2	2	2	2
+6	+16	+26	+36	+46	+56	+66	+76	+86	+96
<hr/> 8	<hr/> 18	<hr/> 28	<hr/> 38	<hr/> 48	<hr/> 58	<hr/> 68	<hr/> 78	<hr/> 88	<hr/> 98

## SIXTH COMBINATION

2	2	2	2	2	2	2	2	2	2
+7	+17	+27	+37	+47	+57	+67	+77	+87	+97
<hr/> 9	<hr/> 19	<hr/> 29	<hr/> 39	<hr/> 49	<hr/> 59	<hr/> 69	<hr/> 79	<hr/> 89	<hr/> 99



## SEVENTH COMBINATION

2	2	2	2	2	2	2	2	2	2
+8	+18	+28	+38	+48	+58	+68	+78	+88	+98
<u>10</u>	<u>20</u>	<u>30</u>	<u>40</u>	<u>50</u>	<u>60</u>	<u>70</u>	<u>80</u>	<u>90</u>	<u>100</u>

## EIGHTH COMBINATION

2	2	2	2	2	2	2	2	2	2
+9	+19	+29	+39	+49	+59	+69	+79	+89	+99
<u>11</u>	<u>21</u>	<u>31</u>	<u>41</u>	<u>51</u>	<u>61</u>	<u>71</u>	<u>81</u>	<u>91</u>	<u>101</u>

## GROUP (c)

## FIRST COMBINATION

3	3	3	3	3	3	3	3	3	3
+3	+13	+23	+33	+43	+53	+63	+73	+83	+93
<u>6</u>	<u>16</u>	<u>26</u>	<u>36</u>	<u>46</u>	<u>56</u>	<u>66</u>	<u>76</u>	<u>86</u>	<u>96</u>

## SECOND COMBINATION

4	4	4	4	4	4	4	4	4	4
+4	+14	+24	+34	+44	+54	+64	+74	+84	+94
<u>8</u>	<u>18</u>	<u>28</u>	<u>38</u>	<u>48</u>	<u>58</u>	<u>68</u>	<u>78</u>	<u>88</u>	<u>98</u>

## THIRD COMBINATION

5	5	5	5	5	5	5	5	5	5
+5	+15	+25	+35	+45	+55	+65	+75	+85	+95
<u>10</u>	<u>20</u>	<u>30</u>	<u>40</u>	<u>50</u>	<u>60</u>	<u>70</u>	<u>80</u>	<u>90</u>	<u>100</u>

## FOURTH COMBINATION

6	6	6	6	6	6	6	6	6	6
+6	+16	+26	+36	+46	+56	+66	+76	+86	+96
<u>12</u>	<u>22</u>	<u>32</u>	<u>42</u>	<u>52</u>	<u>62</u>	<u>72</u>	<u>82</u>	<u>92</u>	<u>102</u>

## FIFTH COMBINATION

7	7	7	7	7	7	7	7	7	7
+7	+17	+27	+37	+47	+57	+67	+77	+87	+97
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
14	24	34	44	54	64	74	84	94	104

## SIXTH COMBINATION

8	8	8	8	8	8	8	8	8	8
+8	+18	+28	+38	+48	+58	+68	+78	+88	+98
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
16	26	36	46	56	66	76	86	96	106

## SEVENTH COMBINATION

9	9	9	9	9	9	9	9	9	9
+9	+19	+29	+39	+49	+59	+69	+79	+89	+99
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
18	28	38	48	58	68	78	88	98	108

## PROBLEMS

## FIRST WEEK

1. Show pupils how to use the multiplication table in solving concrete problems in multiplication.

(a) *Preparation.* Have the table used in this lesson placed on the blackboard.

(b) *Problem.* A tailor made 8 vests. On each vest he placed 6 buttons. How many buttons did he use?

This problem should be announced orally. When the expression "8 vests" is reached it may be written

on the blackboard. The expression "6 buttons" may be similarly treated.

(c) *Exercise.*

*Teacher.*—"What would you do first, Jane?"

*Jane.*—"Count out six buttons from the supply."

*Teacher.*—"How many times would you do this, May?"

*May.*—"Eight times."

*Teacher.*—"Who can write the story that far?"

After hands are shown, a pupil is sent to the board. He writes  $8 \times 6 =$

*Teacher.*—"You may all write the story that far."

Pupils do so.

*Teacher.*—"Let us finish the story from the table, instead of by counting. John, take a pointer, move it downward from the top of the second column until you come to  $8 \times$ . Then keep your pointer at that place."

John does so.

*Teacher.*—"William, you may take another pointer, place it at the left end of the top row, and move it towards the right until you come to 6. Keep it at that place.

"Now, while John moves his pointer to the right William may move his downward until the pointers meet."

The boys do so.

*Teacher.*—"At what number do they meet, Lucy?"

*Lucy.*—"Forty-eight."

*Teacher.*—"Yes; you may finish the story by writing 48; thus,  $8 \times 6 = 48$ ."

Pupils do so.

*Teacher.*—"Samuel may tell me the story."

Samuel says something equivalent to the following (no set form is insisted on):

"A tailor made 8 vests. On each vest he placed 6 buttons. He used 48 buttons."

### ILLUSTRATION

		1	2	3	4	5	6	7	8	9	10	11	12
$\frac{1}{2}$ of	2 x	2	4	6	8	10	12	14	16	18	20	22	24
$\frac{1}{3}$ of	3 x	3	6	9	12	15	18	21	24	27	30	33	36
$\frac{1}{4}$ of	4 x	4	8	12	16	20	24	28	32	36	40	44	48
$\frac{1}{5}$ of	5 x	5	10	15	20	25	30	35	40	45	50	55	60
$\frac{1}{6}$ of	6 x	6	12	18	24	30	36	42	48	54	60	66	72
$\frac{1}{7}$ of	7 x	7	14	21	28	35	42	49	56	63	70	77	84
$\frac{1}{8}$ of	8 x	8	16	24	32	40	48	56	64	72	80	88	96
$\frac{1}{9}$ of	9 x	9	18	27	36	45	54	63	72	81	90	99	108
$\frac{1}{10}$ of	10 x	10	20	30	40	50	60	70	80	90	100	110	120
$\frac{1}{11}$ of	11 x	11	22	33	44	55	66	77	88	99	110	121	132
$\frac{1}{12}$ of	12 x	12	24	36	48	60	72	84	96	108	120	132	144

JOHN'S POINTER

JOHN'S POINTER

WILLIAM'S POINTER

**Drill**

Finish the following from the table:

$3 \times 4 =$	$5 \times 7 =$	$9 \times 9 =$
$4 \times 8 =$	$3 \times 5 =$	$2 \times 6 =$
$9 \times 3 =$	$2 \times 9 =$	$5 \times 4 =$
$7 \times 5 =$	$6 \times 7 =$	$8 \times 9 =$
$6 \times 2 =$	$4 \times 6 =$	$10 \times 5 =$
$3 \times 9 =$	$8 \times 8 =$	$3 \times 7 =$
$2 \times 5 =$	$7 \times 9 =$	$4 \times 9 =$
$5 \times 8 =$	$6 \times 5 =$	$6 \times 8 =$
$8 \times 3 =$	$10 \times 4 =$	$11 \times 10 =$
$11 \times 11 =$	$12 \times 5 =$	$7 \times 8 =$

2. Each of 6 plates contained 9 candy eggs. How many candy eggs were on all the plates?

3. Edna had 7 photographs taken. 3 of them were poor. How many were good?

4. In Jane's garden are 9 rows of lily bulbs, with 8 lily bulbs in each row. How many lily bulbs are in the garden?

5. There are 9 panes in one window and 4 in another. How many panes are in both windows?

6. John had 8 old books. On Christmas he was given 3 new ones. How many books had he then?

7. There were 15 mice in a barn. A gray cat caught 6 of them. How many mice were left in the barn?

8. Each of 4 boys made 6 snowballs. How many snowballs did they all make?

9. There were 8 boats in each of 4 boathouses. How many boats altogether?

10. It took 7 posts for one fence and 9 for another. How many posts did it take for both fences?

11. There were 5 pupils absent, and 7 times as many present. How many pupils were present?

12. In the morning I counted 15 ships on the river. At noon there were only 7. How many ships had sailed away?

13. Mary has 11 dolls. They are all paper dolls but 4. How many paper dolls has she?

14. There are 5 rooms in a new house. It will take 4 rugs for each room. How many rugs will be needed in the house?

15. George walked 3 blocks and rode 9 blocks. How many blocks did he go?

16. On a track there are 5 cars. Each car has 8 wheels. How many wheels have all the cars?

### Drills

*First Day.* Group (a) and extensions of 1st, 2nd, and 3rd combinations.

*Second Day.* Group (a) and extensions of 4th, 5th, and 6th combinations.

*Third Day.* Group (a) and extensions of 7th, 8th, and 9th combinations.

## SECOND WEEK

1. Show pupils how to use the multiplication table in solving concrete problems in measurement.

(a) *Preparation.* Have the table used in this exercise placed on the blackboard.

(b) *Problem.* A tailor placed 6 buttons on each vest made by him. He used 48 buttons. How many vests did he make?

This problem is to be announced orally, the expressions "6 buttons" and "48 buttons" being written on the blackboard by the teacher as she pronounces them.

(c) *Exercise.*

*Teacher.*—"What would you do first, Mary?"

*Mary.*—"Count 48 buttons from the supply."

*Teacher.*—"What next, Samuel?"

*Samuel.*—"Take 6 buttons from 48, for the first vest."

*Teacher.*—"Then what?"

*Samuel.*—"Keep doing this until the 48 buttons are used."

*Teacher.*—"Who can write the story that far?"

After hands are shown, a pupil is sent to the board. He writes  $6 \overline{)48}$ .

*Teacher.*—"You may all write the story that far." Pupils do so.

*Teacher.*—"Let us try to finish the story from the table instead of by counting."

“Sarah, take a pointer; place it at the left end of the top row.

“Move it towards the right until you come to 6.

“Move it downward from 6 to 48.

## ILLUSTRATION

		1	2	3	4	5	6	7	8	9	10	11	12
$\frac{1}{2}$ of	2 x	2	4	6	8	10	12	14	16	18	20	22	24
$\frac{1}{3}$ of	3 x	3	6	9	12	15	18	21	24	27	30	33	36
$\frac{1}{4}$ of	4 x	4	8	12	16	20	24	28	32	36	40	44	48
$\frac{1}{5}$ of	5 x	5	10	15	20	25	30	35	40	45	50	55	60
$\frac{1}{6}$ of	6 x	6	12	18	24	30	36	42	48	54	60	66	72
$\frac{1}{7}$ of	7 x	7	14	21	28	35	42	49	56	63	70	77	84
$\frac{1}{8}$ of	8 x	8	16	24	32	40	48	56	64	72	80	88	96
$\frac{1}{9}$ of	9 x	9	18	27	36	45	54	63	72	81	90	99	108
$\frac{1}{10}$ of	10 x	10	20	30	40	50	60	70	80	90	100	110	120
$\frac{1}{11}$ of	11 x	11	22	33	44	55	66	77	88	99	110	121	132
$\frac{1}{12}$ of	12 x	12	24	36	48	60	72	84	96	108	120	132	144

“Move it to the left from 48 to the second column.

“What have you there?”

*Sarah.*—“Eight times.”

*Teacher.*—“You may finish the story by writing 8x, thus 6)48”

8x



Pupils do so.

*Teacher.*—"Richard, you may tell the story."

Richard says something equivalent to the following (no set form is insisted on):

"A tailor placed 6 buttons on each vest he made until he had used 48 buttons. He made 8 vests.

### Drill

Finish the following from the table:

2) <u>14</u>	8) <u>56</u>	10) <u>100</u>	7) <u>35</u>	4) <u>24</u>
5) <u>30</u>	3) <u>24</u>	5) <u>45</u>	8) <u>72</u>	8) <u>24</u>
9) <u>63</u>	6) <u>18</u>	9) <u>72</u>	2) <u>16</u>	11) <u>77</u>
7) <u>84</u>	12) <u>108</u>	4) <u>28</u>	3) <u>30</u>	12) <u>60</u>
4) <u>32</u>	3) <u>36</u>	2) <u>8</u>	5) <u>40</u>	9) <u>27</u>

2. 18 mice were caught in a store one night, 6 in each trap. How many traps were used?

3. A gardener had 60 onions and made them into bunches. He put 12 onions in each bunch. How many bunches did he make?

4. There are 24 chapters in a book. A boy reads 3 chapters each evening. How many evenings will it take him to read the book?

5. Alice has 8 pieces of blue ribbon and 7 pieces of red ribbon. How many pieces of ribbon has she?

6. 7 families of spiders live in our currant bushes. If there are 8 spiders in each family, how many spiders in all?

7. An expressman started for the depot with 10 trunks on his wagon. When he reached the depot, he found he had only 8. How many had fallen off?

8. Henry learns 4 lessons a day. How many lessons will he learn in 6 days?

9. John shot a gun 17 times. He hit the target 8 times. How many times did he miss?

10. There are 36 pupils in a room. The teacher formed classes and put 12 pupils in each class. How many classes did she form?

11. A newsboy sold 9 morning papers and 8 evening papers. How many papers did he sell?

12. If each cage holds 6 birds, how many birds will 7 cages hold?

13. There are 12 men in a band. All of them play horns but 3. How many play horns?

14. I gave 42 tops to some boys, giving each boy 7 tops. How many boys were there?

15. A girl has 9 paper dolls, and 3 dresses for each. How many dresses has she for her dolls?

16. A girl picked 40 flowers and made bunches of them. She put 8 flowers in each bunch. How many bunches did she make?

### Drills

*First Day.* Group (b) and extensions of 1st, 2nd, and 3rd combinations.

*Second Day.* Group (b) and extensions of 4th, 5th, and 6th combinations.

*Third Day.* Group (b) and extensions of 7th and 8th combinations.

*Fourth Day.* Miscellaneous.

### THIRD WEEK

1. Show pupils how to use the multiplication table to find results in partition.

(a) *Preparation.* Place upon the blackboard the table given below.

ILLUSTRATION

		1	2	3	4	5	6	7	8	9	10	11	12
$\frac{1}{2}$ of	2 x	2	4	6	8	10	12	14	16	18	20	22	24
$\frac{1}{3}$ of	3 x	3	6	9	12	15	18	21	24	27	30	33	36
$\frac{1}{4}$ of	4 x	4	8	12	16	20	24	28	32	36	40	44	48
$\frac{1}{5}$ of	5 x	5	10	15	20	25	30	35	40	45	50	55	60
$\frac{1}{6}$ of	6 x	6	12	18	24	30	36	42	48	54	60	66	72
$\frac{1}{7}$ of	7 x	7	14	21	28	35	42	49	56	63	70	77	84
$\frac{1}{8}$ of	8 x	8	16	24	32	40	48	56	64	72	80	88	96
$\frac{1}{9}$ of	9 x	9	18	27	36	45	54	63	72	81	90	99	108
$\frac{1}{10}$ of	10 x	10	20	30	40	50	60	70	80	90	100	110	120
$\frac{1}{11}$ of	11 x	11	22	33	44	55	66	77	88	99	110	121	132
$\frac{1}{12}$ of	12 x	12	24	36	48	60	72	84	96	108	120	132	144

(b) *Problem.* A tailor used 48 buttons in making 8 vests. He put the same number of buttons on each vest. How many buttons did he use for a vest?

This problem is to be announced orally by the teacher, the expressions "48 buttons" and "8 vests" being written on the blackboard during the announcement.

(c) *Exercise.*

*Teacher.*—"What would you do first, George?"

*George.*—"Count 48 buttons from the supply."

*Teacher.*—"What next, Charles?"

*Charles.*—"Count out the buttons 'one at a time' in 8 places, until all are placed."

*Teacher.*—"Who can write the story that far?"

After hands are shown, a pupil is sent to the board. He writes  $\frac{1}{8}$  of 48 =

*Teacher.*—"You may all write the story that far."

Pupils do so.

*Teacher.*—"Let us try to finish the story from the table instead of by counting. Susan, take a pointer and place it at the top of the first column.

"Move it downward until you come to  $\frac{1}{8}$ .

"Move it to the right to 48.

"Move it upward from 48 to the top of the column.

"What do you find there?"

*Susan.*—"Six."

*Teacher.*—"You may finish the story by writing 6, thus,  $\frac{1}{8}$  of 48 = 6."

Pupils do so.

*Teacher.*—"Susan, you may tell the story."

Susan says something equivalent to the following (no fixed form is insisted on):

"A tailor used 48 buttons in making 8 vests. He placed 6 on each vest."

### Drill

Finish the following from the table:

$\frac{1}{2}$ of 8 =	$\frac{1}{7}$ of 56 =	$\frac{1}{9}$ of 45 =	$\frac{1}{6}$ of 54 =
$\frac{1}{4}$ of 24 =	$\frac{1}{5}$ of 15 =	$\frac{1}{3}$ of 9 =	$\frac{1}{5}$ of 35 =
$\frac{1}{9}$ of 27 =	$\frac{1}{8}$ of 24 =	$\frac{1}{4}$ of 16 =	$\frac{1}{3}$ of 33 =
$\frac{1}{4}$ of 12 =	$\frac{1}{11}$ of 110 =	$\frac{1}{8}$ of 56 =	$\frac{1}{6}$ of 48 =
$\frac{1}{12}$ of 48 =	$\frac{1}{12}$ of 72 =	$\frac{1}{2}$ of 24 =	$\frac{1}{9}$ of 54 =
$\frac{1}{3}$ of 36 =	$\frac{1}{7}$ of 42 =	$\frac{1}{7}$ of 63 =	$\frac{1}{8}$ of 40 =
$\frac{1}{10}$ of 60 =	$\frac{1}{8}$ of 72 =	$\frac{1}{5}$ of 40 =	

2. A man sold 27 pigeons to 9 boys. Each boy bought the same number. How many pigeons did each boy buy?

3. Harold made 4 kites of the same size. He used altogether 32 tacks. How many tacks were used for each kite?

4. A postman took 9 letters from each of 4 boxes. How many letters did he take from all the boxes?

5. From my window I can see 8 poplar trees and 7 elms. How many trees can I see?

6. A boy has 11 rabbits. 6 of them are white and

the others are gray. How many gray rabbits are there?

7. A lady made 21 aprons for her 3 little girls. How many aprons could each girl have?

8. John made 5 piles of his blocks. There were 9 blocks in each pile. How many blocks had he?

9. A lady sewed 35 buttons on some waists. She put 5 buttons on each waist. How many waists were there?

10. 20 apples were used in making 5 pies of the same size. How many apples did it take for a pie?

11. 24 clocks filled 8 boxes of the same size. How many clocks were put into each box?

12. Our teacher gave a drill in the hall. She had 5 rows of pupils with 8 in each row. How many pupils were there in the drill?

13. May bought 24 pencils, 6 in a box. How many boxes of pencils did she buy?

14. A lady baked 18 pies and put them in the pantry, 6 on a shelf. How many shelves did she use?

15. On a shelf were 14 books. 9 of them had cloth covers. The others had paper covers. How many had paper covers?

16. A man divided 56 pencils among some children. He gave each child 8 pencils. How many children were there?

### Drills

*First Day.* Group (c) and extensions of the 1st, 2nd, and 3rd combinations.

*Second Day.* Group (c) and extensions of the 4th and 5th combinations.

*Third Day.* Group (c) and extensions of the 6th and 7th combinations.

*Fourth Day.* Groups (a), (b), and (c) with miscellaneous extensions.

#### FOURTH WEEK

1. Show pupils how to add a column, as follows:

*Problem.* Nellie had 6 pieces of red chalk, 2 pieces of blue chalk, and 4 pieces of white chalk. How many pieces of chalk did she have?

Imagined counting shows the process to be addition. Therefore write the numbers one under the other so as to form a column. Let the number that appears first in the problem (6) be written at the top of the column. Below it write the number appearing next in the problem (2); and at the bottom of the column write 4, the number appearing last in the problem. That is, the column is *written downwards*, and when finished appears thus, 6

$$\begin{array}{r} 6 \\ 2 \\ +4 \\ \hline \end{array}$$

The column though *written downwards* is to be *added upwards*, as follows:

4 and 2 are 6. (See group (b), 3rd combination.)

6 and 6 are 12. (See group (c), 4th combination.)

Hence the sum is 12, which is to be written below the line, giving the completed numerical expression,

$$\begin{array}{r} 6 \\ 2 \\ +4 \\ \hline 12 \end{array}$$

It is to be remembered that all columns derived from concrete problems are constructed by writing downwards as described above, and that all columns, however obtained, are to be added upwards.

### Drill

Add the following:

2	2	8	2	7	2	3	1	2	2	5
1	1	1	2	2	1	1	2	2	1	2
<u>+2</u>	<u>+3</u>	<u>+7</u>	<u>+7</u>	<u>+5</u>	<u>+8</u>	<u>+2</u>	<u>+7</u>	<u>+4</u>	<u>+6</u>	<u>+3</u>

2. In a street car there were 2 women, 2 children, and 4 men. How many people were in the car?

3. A lady had in her desk 15 envelopes. 6 of them were white and the others yellow. How many yellow envelopes had she?

4. Ethel found 6 eggs in one nest, 2 in another, and 4 in another. How many eggs did she find?

5. The teacher wrote 32 words on the board. She put the words in 4 equal rows. How many words were in each row?

6. Harry had two books. There were 6 pictures



in one book and 16 in the other. How many pictures were in both books? (See group (c), 4th combination, 1st extension, page 133.)

7. We go to school 5 days a week. We have 4 new spelling words each day. How many new spelling words do we have in a week?

8. A lady answered 9 telephone calls a day. In how many days did she answer 45 calls?

9. A woman sewed 42 buttons on 7 pairs of gloves. How many buttons was that for each pair?

10. A school has 24 boys and 31 girls. How many pupils in the school?

11. A blacksmith shod 7 horses in a day. He put 4 horseshoes on each horse. How many horseshoes did he fit in a day?

12. There were 14 ripe cherries on Willie's cherry tree. The birds ate 6. How many were left?

13. John had 2 flags, William had 4, and Henry had 4. How many flags had all the boys? (Group (c), 2nd combination, and group (b), 7th combination.)

14. 4 boys had altogether 28 ducks. All of the boys had the same number. How many ducks did the boys have apiece?

15. Bessie spelled 6 words wrong and 16 right. How many words did she spell? (See group (c), 4th combination, 1st extension.)

16. 14 eggs were under a sitting hen. 9 chickens were hatched. How many of the eggs were bad?

**Drills**

*First Day.* Group (a) and its extensions.

*Second Day.* Group (b) and its extensions.

*Third Day.* Group (c) and its extensions.

*Fourth Day.* Miscellaneous.

**FIFTH WEEK**

1. Develop further the addition of a column, as follows:

*Problem.* On one shelf there were 2 books; on another shelf, 8 books; and on another shelf, 8 books. How many books were on the three shelves?

*Solution.* Imagined counting gives the following numerical expression, 2

$$\begin{array}{r} 2 \\ 8 \\ +8 \\ \hline \end{array}$$

Commencing at the bottom, 8 and 8 are 16 (group (c), 6th combination). 16 and 2 are 18 (group (b), 5th combination, 1st extension). Hence the sum is 18, which is to be written below the line, giving the completed numerical expression, 2

$$\begin{array}{r} 2 \\ 8 \\ +8 \\ \hline 18 \end{array}$$

(See pages 132 and 134.)

## Drill

1	6	2	2	1	1	2	2	2
8	3	4	2	9	1	9	6	2
4	1	2	8	2	5	1	3	6
<u>+4</u>	<u>+2</u>	<u>+2</u>	<u>+8</u>	<u>+7</u>	<u>+5</u>	<u>+8</u>	<u>+3</u>	<u>+6</u>

1	2	2	7	2	2	1	9	2
7	2	1	1	5	1	5	9	5
2	1	2	3	5	9	1	1	9
<u>+5</u>	<u>+3</u>	<u>+3</u>	<u>+3</u>	<u>+5</u>	<u>+9</u>	<u>+9</u>	<u>+1</u>	<u>+2</u>

2. Mary had 9 paper dolls; Ellen had 8. How many paper dolls had both girls?

3. On one dogwood branch there were 7 blossoms. On another branch there were only 2 blossoms. On still another branch there were 9 blossoms. How many blossoms were on all three branches?

4. A family bought 12 papers a week for 7 weeks. How many papers did they buy in that time?

5. A little girl picked 14 bunches of pansies. She sold all of them but 5 bunches. How many bunches did she sell?

6. 7 girls have 63 doll dresses. How many has each girl, if all have the same number?

7. At a store were 21 boxes of strawberries. Some customers bought 3 boxes each. How many customers were supplied, if all the strawberries were sold?

8. 2 of the children's Easter eggs were red, 7 were

purple, and 7 were blue. How many Easter eggs had the children?

9. A family used 24 pints of cider in 8 days, using the same number of pints each day. How much was that a day?

10. Bessie, Fay, and Mary each had 7 pieces of candy. How many pieces had they altogether?

11. Two girls have together 14 buttons. One girl has 8 buttons. How many has the other?

12. George showed his little brother some picture books. There were 6 pictures in each book and 24 pictures altogether. How many picture books were there?

13. A party of girls made 48 soap bubbles. They made 8 bubbles apiece. How many girls were in the party?

14. There were 7 broken panes in one window, 2 in another, and 9 in another. How many panes were broken in the three windows?

15. There were 12 oranges in a paper sack. 8 of them were sweet and the others sour. How many sour oranges were in the sack?

16. 5 boys went out to fly their kites. Each of them had 3 kites. How many kites did they all have?

#### Drills

<i>First Day.</i>	1	6	2	9	2
	2	5	8	2	6
	3	3	7	7	8
	<u>+3</u>	<u>+2</u>	<u>+1</u>	<u>+2</u>	<u>+8</u>

<i>Second Day.</i>	2	2	4	2	9
	8	5	2	1	1
	4	5	6	6	9
	<u>+4</u>	<u>+5</u>	<u>+6</u>	<u>+2</u>	<u>+9</u>

<i>Third Day.</i>	5	7	8	6	4
	5	2	2	8	3
	8	4	5	7	9
	<u>+2</u>	<u>+1</u>	<u>+1</u>	<u>+1</u>	<u>+2</u>

<i>Fourth Day.</i>	6	8	7	2	2
	8	9	2	1	1
	9	6	1	7	9
	<u>+9</u>	<u>+6</u>	<u>+8</u>	<u>+7</u>	<u>+9</u>

## SIXTH WEEK

1. There were 6 birds on a fence, 5 on a tree, and 1 on the ground. How many birds altogether?

2. A lady planted 14 rose bushes. They all died but 5. How many died?

3. A carpenter built 7 houses. He put 9 windows in each house. How many windows did he make in all?

4. One girl made 6 cakes; another girl made 2 cakes; and another girl made 4 cakes. How many cakes did the girls make altogether?

5. Mary put 18 spools into 3 little boxes, putting

the same number into each box. How many spools did she put into each box?

6. 24 roses were placed in vases. There were 8 roses in each vase. How many vases were used?

7. James had 2 tops and bought 6 more. William had 2 tops and bought 4 more. How many tops did the boys then have together?

8. 45 books belonged to 9 pupils. All of the pupils had the same number of books. How many books had each pupil?

9. There were 15 birds on a fence. How many were left after 8 flew away?

10. 6 boys had 5 tops apiece. How many tops did they all have?

11. Bessie's mother made 16 cakes and gave the children 9 of them. How many cakes were left?

12. George had 2 marbles in one bag, 7 in another bag, and 7 in still another bag. How many marbles had he?

13. There were 7 cars on a track, and they had 56 wheels altogether. All the cars had the same number of wheels. How many wheels were on each car?

14. 3 little girls had 8 roses apiece. How many roses had they all?

15. A lady has 7 cards. There are 12 buttons on each card. How many buttons has she?

16. Some boys had 7 ducks apiece and 28 ducks altogether. How many ducks were there?

**Drills***First Day.* (1) Group (c) and subtractions.

(2)	1	2	9	2
	2	8	5	2
	5	8	2	7
	<u>+5</u>	<u>+2</u>	<u>+3</u>	<u>+7</u>

*Second Day.* (1) Group (b) and subtractions.

(2)	2	4	6	7
	2	7	8	7
	9	2	6	8
	<u>+9</u>	<u>+5</u>	<u>+2</u>	<u>+2</u>

*Third Day.* (1) Group (a) and subtractions.

(2)	8	8	7	9
	4	5	6	2
	3	4	5	6
	<u>+1</u>	<u>+1</u>	<u>+1</u>	<u>+1</u>

*Fourth Day.* Miscellaneous.**PART XI**

I and II. Read and write numbers to 500.

III. Solve problems as described below.

It is to be remembered:

1. That problems are to be announced as

wholes. The practice of writing the numbers used in the problem as soon as they are announced should be continued. As soon as the pupil's readiness in reading script justifies it, the teacher should occasionally write on the blackboard a very short problem without any oral announcement by the teacher. Have it read by pupils with proper expression; underscore the numbers that they may be easily found; then let it be solved.

2. That the modes of counting are:
  - (a) Additive.
  - (b) Subtractive.
  - (c) Partitive.
3. That the numerical expressions employed are:
  - (a) Additive.
  - (b) Subtractive.
  - (c) Partitive.
  - (d) Multiplicative.
  - (e) Measurement.
4. That in all operations the numerical expressions are suggested by imagined counting. That in addition and subtraction the results are obtained by computation. That in multiplication, measurement, and partition, results are obtained by reference to the tables.



IV. 1. Review the addition table and the extensions of groups (a), (b), and (c).

2. Extend groups (d), (e), and (f) as follows:

### GROUP (d)

#### FIRST COMBINATION

4	4	4	4	4	4	4	4	4	4
+3	+13	+23	+33	+43	+53	+63	+73	+83	+93
<u>7</u>	<u>17</u>	<u>27</u>	<u>37</u>	<u>47</u>	<u>57</u>	<u>67</u>	<u>77</u>	<u>87</u>	<u>97</u>

#### SECOND COMBINATION

5	5	5	5	5	5	5	5	5	5
+4	+14	+24	+34	+44	+54	+64	+74	+84	+94
<u>9</u>	<u>19</u>	<u>29</u>	<u>39</u>	<u>49</u>	<u>59</u>	<u>69</u>	<u>79</u>	<u>89</u>	<u>99</u>

#### THIRD COMBINATION

6	6	6	6	6	6	6	6	6	6
+5	+15	+25	+35	+45	+55	+65	+75	+85	+95
<u>11</u>	<u>21</u>	<u>31</u>	<u>41</u>	<u>51</u>	<u>61</u>	<u>71</u>	<u>81</u>	<u>91</u>	<u>101</u>

#### FOURTH COMBINATION

7	7	7	7	7	7	7	7	7	7
+6	+16	+26	+36	+46	+56	+66	+76	+86	+96
<u>13</u>	<u>23</u>	<u>33</u>	<u>43</u>	<u>53</u>	<u>63</u>	<u>73</u>	<u>83</u>	<u>93</u>	<u>103</u>

#### FIFTH COMBINATION

8	8	8	8	8	8	8	8	8	8
+7	+17	+27	+37	+47	+57	+67	+77	+87	+97
<u>15</u>	<u>25</u>	<u>35</u>	<u>45</u>	<u>55</u>	<u>65</u>	<u>75</u>	<u>85</u>	<u>95</u>	<u>105</u>

## SIXTH COMBINATION

9	9	9	9	9	9	9	9	9	9
+8	+18	+28	+38	+48	+58	+68	+78	+88	+98
<u>17</u>	<u>27</u>	<u>37</u>	<u>47</u>	<u>57</u>	<u>67</u>	<u>77</u>	<u>87</u>	<u>97</u>	<u>107</u>

## GROUP (e)

## FIRST COMBINATION

9	9	9	9	9	9	9	9	9	9
+3	+13	+23	+33	+43	+53	+63	+73	+83	+93
<u>12</u>	<u>22</u>	<u>32</u>	<u>42</u>	<u>52</u>	<u>62</u>	<u>72</u>	<u>82</u>	<u>92</u>	<u>102</u>

## SECOND COMBINATION

9	9	9	9	9	9	9	9	9	9
+4	+14	+24	+34	+44	+54	+64	+74	+84	+94
<u>13</u>	<u>23</u>	<u>33</u>	<u>43</u>	<u>53</u>	<u>63</u>	<u>73</u>	<u>83</u>	<u>93</u>	<u>103</u>

## THIRD COMBINATION

9	9	9	9	9	9	9	9	9	9
+5	+15	+25	+35	+45	+55	+65	+75	+85	+95
<u>14</u>	<u>24</u>	<u>34</u>	<u>44</u>	<u>54</u>	<u>64</u>	<u>74</u>	<u>84</u>	<u>94</u>	<u>104</u>

## FOURTH COMBINATION

9	9	9	9	9	9	9	9	9	9
+6	+16	+26	+36	+46	+56	+66	+76	+86	+96
<u>15</u>	<u>25</u>	<u>35</u>	<u>45</u>	<u>55</u>	<u>65</u>	<u>75</u>	<u>85</u>	<u>95</u>	<u>105</u>

## FIFTH COMBINATION

9	9	9	9	9	9	9	9	9	9
+7	+17	+27	+37	+47	+57	+67	+77	+87	+97
<u>16</u>	<u>26</u>	<u>36</u>	<u>46</u>	<u>56</u>	<u>66</u>	<u>76</u>	<u>86</u>	<u>96</u>	<u>106</u>

## GROUP (f)

## FIRST COMBINATION

8	8	8	8	8	8	8	8	8	8
+3	+13	+23	+33	+43	+53	+63	+73	+83	+93
<hr/> 11	<hr/> 21	<hr/> 31	<hr/> 41	<hr/> 51	<hr/> 61	<hr/> 71	<hr/> 81	<hr/> 91	<hr/> 101

## SECOND COMBINATION

8	8	8	8	8	8	8	8	8	8
+4	+14	+24	+34	+44	+54	+64	+74	+84	+94
<hr/> 12	<hr/> 22	<hr/> 32	<hr/> 42	<hr/> 52	<hr/> 62	<hr/> 72	<hr/> 82	<hr/> 92	<hr/> 102

## THIRD COMBINATION

8	8	8	8	8	8	8	8	8	8
+5	+15	+25	+35	+45	+55	+65	+75	+85	+95
<hr/> 13	<hr/> 23	<hr/> 33	<hr/> 43	<hr/> 53	<hr/> 63	<hr/> 73	<hr/> 83	<hr/> 93	<hr/> 103

## FOURTH COMBINATION

8	8	8	8	8	8	8	8	8	8
+6	+16	+26	+36	+46	+56	+66	+76	+86	+96
<hr/> 14	<hr/> 24	<hr/> 34	<hr/> 44	<hr/> 54	<hr/> 64	<hr/> 74	<hr/> 84	<hr/> 94	<hr/> 104

3. Subtract large numbers without carrying.
  4. Add a single column.
  5. Carrying in addition.
- V. Difficulties in concrete problems.
1. Introduction of artificial pluralities.
    - (a) Counting pints. (See page 160.)
    - (b) Counting quarts. (See page 163.)
    - (c) Counting gallons. (See page 166.)

## COUNTING PINTS

## FIRST WEEK

1. Show pupils how to count pints, as follows:

Up to this point the problems solved by pupils have dealt with natural pluralities. That is, the units were naturally separated from one another, so that their number could be ascertained by simple counting. Now we come to deal with capacity, a property not naturally a plurality, but which may be thought of as such by the use of artificial units, the number of which can be ascertained by measuring or counting with the help of a standard of some kind.

In this exercise the artificial unit is the *pint*; and the standard or instrument which enables us to count these units is the *pint measure*.

Upon a table are placed a pint measure, a can filled with water, and an empty kettle or small tub.

Teacher lifts the pint measure and says: "This is a pint measure. If I fill it with milk, I will have a pint of milk; if with water, I will have a pint of water. How many pints of water do you think are in this can?"

After various estimates, one of the pupils, Willie, is called to the table to count the pints in the can.

With no more help from the teacher than is necessary, Willie fills the pint measure from the can and empties it into the tub. He repeats this until all the

water is transferred from the can to the tub, counting (aloud) the pints as he pours them out. Meantime, the teacher is moving from the supply a counter for each pint. When Willie has finished, he says, "There were 9 pints of water in the can."

Teacher repeats, "Yes, there were 9." (Pointing to the 9 counters.)

### THE FIRST ARTIFICIAL PLURALITY



**KEY.** 1. The quart measure. 2. The gallon measure. 3. The can. 4. The tub. 5. Willie. 6. The teacher. 7. A supply of counters. 8. The counters already taken from supply. 9. The last one being removed.

### Drill

Have pupils either use counters or describe the counting for each of the following:

(a) There are 2 pints of milk in one can, 5 in

another, 7 in another, and 9 in another. How many pints were in all the cans?

(b) I have 15 pints of root beer, and wish to put it into 5 bottles, putting the same number of pints into each. How many pints will each bottle contain?

(c) Each of 4 jugs held 8 pints of vinegar. How many pints of vinegar were in all the jugs?

(d) There are 14 pints of milk in a can. How many pints remained after 6 were sold?

(e) How many times can 4 pints of oil be taken from a can which holds 36 pints?

2. There were 17 pints of oil in a can. After 8 pints had been used, how many pints were left?

3. A gallon measure will hold 8 pints of oil. It was filled 6 times, and each time emptied into a can. How many pints of oil were poured into the can?

4. A little pear tree has 32 leaves on 4 branches. How many leaves are on each branch, if they have the same number apiece?

5. Edward writes 8 lines in his copybook each day. How many days will it take him to write 40 lines?

6. To how many customers can 4 pints of ice cream apiece be sold from a can containing 20 pints?

7. Florence made 4 rows of picture cards on the table. She put 9 cards in each row. How many picture cards did she use?

8. 15 pints of milk filled 5 pitchers of the same size. How much did each pitcher hold?

9. A family bought 8 pints of cream one week, 7 the next, and 9 the next. How many pints did they buy in the three weeks?

10. A conductor rang up 22 fares on one trip and 26 on another. How many did he ring up on both trips?

11. Mary's mother bought 12 oranges and put them on a shelf in the pantry. 5 of them were used for dinner. How many were left on the pantry shelf?

12. A lady used 32 eggs making cakes. She put 4 eggs in each cake. How many cakes did she make?

13. A boy had 9 white marbles, 7 brown marbles, and 6 glass marbles. How many marbles did he have altogether?

### Drill

*First Day.* Extensions of group (d) with variations.

*Second Day.* Extensions of group (e) with variations.

*Third Day.* Extensions of group (f) with variations.

*Fourth Day.* Miscellaneous.

### SECOND WEEK

1. Examine the exercise on page 160 entitled "Counting Pints," and repeat it, using a quart measure instead of a pint. Incidentally, the fact may be brought out that  $2 \text{ pints} = 1 \text{ quart}$ , but there is no

occasion to memorize it, and it certainly should not be used as an unstated condition of problems. At present all the conditions of the problem should be clearly stated in the simplest language.

### Drill

Have pupils either use counters or describe the counting in solving each of the following:

(a) From 132 quarts how many times can 11 quarts be taken?

(b) There are 7 quarts of water in one bucket, 6 in another, and 5 in another. How much water is in the three buckets?

(c) Each of 9 cans contained 5 quarts of molasses. How many quarts were in all the cans?

(d) A can holding 32 quarts of oil was emptied in 8 days. How much was used each day, if the same number of quarts was used each day?

2. In an orchard were 56 trees, planted 8 in a row. How many rows were there?

3. In an orchard were 9 pear trees, 7 prune trees, and 6 apple trees. How many trees were in the orchard?

4. A milkman had a customer charged with 16 pints of cream. The customer took 2 pints a day. How many days had the bill been running?

5. Two boys picked 15 quarts of cherries. But 6 quarts of them were unfit to use. How many quarts of good cherries did they pick?



6. 3 street cars just passed our house. There were 12 passengers in each of them. How many passengers were in all the cars?

7. We have 45 rosebushes in 5 equal hedges. How many bushes in each hedge?

8. William had 8 black tops, 6 red ones, and 5 white ones. How many tops had he altogether?

9. A keg contained 18 quarts of vinegar. After 9 quarts had been used, how much vinegar remained in the keg?

10. In one box there were 9 pencils, in another 7, and in another 6. How many pencils were in the three boxes?

11. A horse was fed 3 quarts of oats a day. How many days did 24 quarts last him?

12. Each of 9 robins ate 7 cherries from George's tree. How many cherries did all the robins eat?

13. From a 15-pint can of paint 7 pints were used. How many pints remained in the can?

14. In two days a teamster fed 5 quarts of oats to each of his 4 horses. How many quarts did he feed to all of them?

### Drill

Add the following:

<i>First Day.</i>	2	2	1	9	8	2
	7	8	2	2	1	3
	4	6	8	9	5	7
	<u>+3</u>	<u>+5</u>	<u>+7</u>	<u>+8</u>	<u>+4</u>	<u>+6</u>

<i>Second Day.</i>	8	6	5	7	4	2
	1	1	2	6	2	1
	8	7	6	5	4	3
	<u>+9</u>	<u>+8</u>	<u>+7</u>	<u>+6</u>	<u>+5</u>	<u>+4</u>

<i>Third Day.</i>	1	2	2	8	2
	2	4	4	6	5
	9	9	9	9	9
	<u>+3</u>	<u>+4</u>	<u>+5</u>	<u>+6</u>	<u>+7</u>

<i>Fourth Day.</i>	4	5	9	8	9
	2	1	4	2	7
	3	4	5	6	7
	<u>+9</u>	<u>+9</u>	<u>+9</u>	<u>+9</u>	<u>+9</u>

## THIRD WEEK

1. Show pupils how to count gallons. The mode of proceeding is the same as for pints.

## Drill

Have pupils either use counters or imagine the counting in solving the following:

(a) From a 12-gallon can of coal oil, 5 gallons were used. How many gallons remained in the can?

(b)  $\frac{1}{3}$  of 60 gallons = ?

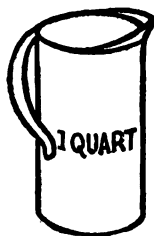
(c) How many gallons of vinegar are in 8 kegs, if 12 gallons are in each keg?

(d) How many days would 12 gallons of coal oil last, if 3 gallons were used each day?

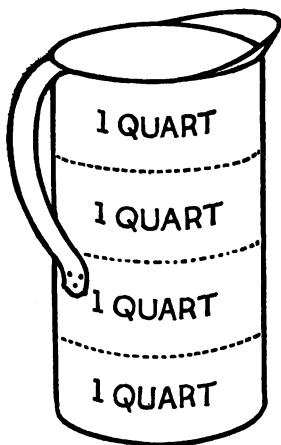
2. 12 quarts of molasses were poured into pitchers holding 2 quarts each. How many pitchers were used?

3. A barrel contained 14 gallons of vinegar. After 9 gallons were used, how many gallons remained in the barrel?

4. In one tank there are 9 gallons of oil. In another tank, there are 12 gallons. How many gallons in both tanks?



QUART



GALLON

5. A painter used 32 panes of glass. He put 8 panes in each window. How many windows did he make?

6. From a can holding 15 pints of ice cream, 9 pints were used. How many pints of ice cream remained in the can?

7. Mary gathered 43 flowers, and Lily gathered 23 flowers. How many flowers did the two girls gather?

8. A druggist bought 4 kegs of lime water, each containing 8 gallons. How many gallons of lime water did he buy?

9. A lady had 14 jars of fruit in her pantry. She used 2 jars each week. How many weeks did the fruit last?

10. John picked 7 quarts of cherries. Ann picked 8 quarts. Tom picked 9 quarts. How many quarts of cherries did they all pick?

11. 6 pitchers, each holding 3 pints of water, were emptied into a bucket. How many pints of water were poured into the bucket?

12. How many 5-gallon cans can be filled from 55 gallons of oil?

13. In a snowball fight, there were 9 boys on one side, 8 boys on the other, and 6 boys looking on. How many boys were present?

14. There are 17 boys in a class. Yesterday 8 of them were absent. How many were present?

### Drill

Add the following:

<i>First Day.</i>	4	5	2	7	2	3
	1	3	1	4	5	1
	8	8	8	8	3	4
	<u>+3</u>	<u>+4</u>	<u>+5</u>	<u>+6</u>	<u>+8</u>	<u>+8</u>

<i>Second Day.</i>	2	2	1	8	3	9
	8	9	9	8	6	2
	5	6	3	4	5	8
	<u>+8</u>	<u>+8</u>	<u>+2</u>	<u>+2</u>	<u>+2</u>	<u>+8</u>

<i>Third Day.</i>	9	4	9	6	2	8
	5	4	2	5	9	4
	6	7	8	9	2	3
	<u>+2</u>	<u>+2</u>	<u>+2</u>	<u>+2</u>	<u>+1</u>	<u>+1</u>

<i>Fourth Day.</i>	6	9	2	9	9	6
	4	5	9	5	9	6
	4	5	6	7	8	9
	<u>+1</u>	<u>+1</u>	<u>+1</u>	<u>+1</u>	<u>+1</u>	<u>+1</u>

## FOURTH WEEK

1. Show pupils how to read and write numbers to 500.

If pupils can read and write numbers to 400, there is nothing new in this exercise, except the use of the figure 5 in the hundreds' place.

The numbers 401, 402, 403, 404, 405, 406, 407, 408, 409 must be frequently read and written.

2. In one month John was absent from school 6 days, Henry 6 days, and Walter 9 days. How many absent marks did the three boys receive?

3. Mary had three dolls. One of them had 6 dresses, another 9 dresses, and another 8 dresses. How many dresses had the three dolls?

4. Jane has 30 doll dresses for her 6 dolls. How many is that apiece, if all have the same number?

5. A cow gave 18 pints of milk. 2 pints were sold to each customer. How many customers were supplied when the milk was all sold?

6. A boy had 5 bags with 9 marbles in each. How many marbles had he in all the bags?

7. On Christmas night Mary had 9 candles at each window of their front room. There were 4 windows in the room. How many candles did she use?

8. A tailor put 8 buttons on each overcoat he made. He used 24 buttons. How many overcoats did he make?

9. A lady preserved 9 pints of blackberries, 7 pints of strawberries, and 8 pints of huckleberries. How many pints of berries did she preserve?

10. There were 15 children in our arithmetic class, but 6 of them left school. How many are now in the class?

11. A barrel contained 17 gallons of vinegar. After 9 gallons were used, how many gallons remained?

12. 7 buckets of water were poured into an empty tub. Each bucket held 5 gallons. How many gallons of water were poured into the tub?

13. 36 pupils in a class marched 4 in each rank.  
How many ranks were there?

14. There are 8 tables in an ice cream parlor. 4  
persons are eating ice cream at each table. How  
many persons are eating ice cream in the parlor?

### Drill

Add the following:

<i>First Day.</i>	6	8	9	7	5	8
	3	6	5	8	7	4
	2	3	4	5	6	7
	<u>+2</u>	<u>+3</u>	<u>+4</u>	<u>+5</u>	<u>+6</u>	<u>+7</u>

<i>Second Day.</i>	2	1	6	3	7	9
	5	8	9	4	6	8
	8	9	3	4	5	6
	<u>+8</u>	<u>+9</u>	<u>+4</u>	<u>+5</u>	<u>+6</u>	<u>+7</u>

<i>Third Day.</i>	6	4	3	6	2	5
	5	8	2	3	4	9
	7	8	3	4	5	4
	<u>+8</u>	<u>+9</u>	<u>+9</u>	<u>+9</u>	<u>+9</u>	<u>+2</u>

<i>Fourth Day.</i>	7	2	6	6	9	8
	2	8	6	9	2	4
	6	7	3	4	5	6
	<u>+9</u>	<u>+9</u>	<u>+8</u>	<u>+8</u>	<u>+8</u>	<u>+8</u>

## FIFTH WEEK

1. Show pupils how to carry in addition, as follows:

*Problem.* In one book there were 28 pictures; in another 77 pictures; in still another 44 pictures; and in a fourth book 32 pictures. How many pictures were in the four books?

Imagined counting gives this numerical expression, 28

77

44

+32

181

Adding the first or right-hand column, we obtain 21.

The right-hand figure, or 1 unit, is written under the units' column, while the 2 tens are carried to the tens' column.

Adding the tens' column, we obtained 18 tens, to be written under the tens' column.

Therefore, there were 181 pictures in the four books.

2. How many balls are there in 3 Roman candles, each containing 8 balls?

3. A milkman sold 29 quarts of milk on one street; 15 quarts on another; and 18 quarts on another. How many quarts did he sell on the three streets?



4. Each of the 9 pupils in our class spelled 6 words. How many words did they all spell?

5. Lucy found 20 eggs in 5 nests. How many did she find in each, if all had the same number?

6. One barrel contained 28 gallons of rain water; another contained 24 gallons; a third 29 gallons. How many gallons of rain water did the three barrels hold?

7. There were 325 gallons of water in a tank. 113 gallons were used. How many gallons were left in the tank?

8. A baker made 24 lemon pies, 27 peach pies, and 28 prune pies. How many pies did he make altogether?

9. 27 boys were divided into 3 ball teams. How many boys were in each team?

10. A boy had 38 rabbits; 21 of them ran away. How many rabbits had the boy left?

11. A boy gathered 26 eggs one day; 22 the next day; 19 the next; and 24 on the fourth day. How many eggs did he gather in the four days?

12. Each of 7 bottles held 2 pints of ink. How much ink was in all the bottles?

13. Samuel picked 28 plums from 4 trees. He picked the same number from each tree. How many did he take from each tree?

14. A boy caught 16 fish the first day, 12 the second day, 9 the third day, and 14 on the fourth day. How many fish did he catch in all?

## Drill

Add the following:

<i>First Day.</i>	6	6	5	5	6	9
	5	3	2	2	8	7
	1	2	3	4	9	8
	<u>+1</u>	<u>+2</u>	<u>+3</u>	<u>+3</u>	<u>+3</u>	<u>+3</u>

<i>Second Day.</i>	6	4	9	8	5	8
	4	5	3	9	8	7
	1	2	4	5	9	8
	<u>+2</u>	<u>+3</u>	<u>+4</u>	<u>+4</u>	<u>+4</u>	<u>+4</u>

<i>Third Day.</i>	4	9	4	8	6	8
	5	7	8	7	3	9
	1	2	5	6	9	8
	<u>+3</u>	<u>+4</u>	<u>+5</u>	<u>+5</u>	<u>+5</u>	<u>+5</u>

<i>Fourth Day.</i>	2	3	4	5	6	7
	6	6	2	8	4	3
	1	2	6	7	9	8
	<u>+4</u>	<u>+5</u>	<u>+6</u>	<u>+6</u>	<u>+6</u>	<u>+6</u>

## SIXTH WEEK

1. On his birthday a little boy received 63 pennies, 7 from each of his relatives. How many relatives gave him pennies?

2. On a pear tree were 15 pears. But the wind blew off 6. How many were left on the tree?

3. There were 6 little girls at Helen's party. She gave each of them 5 pieces of candy. How many pieces of candy did it take?

4. On a teacher's desk were three boxes of chalk. One held 25 pieces; another 16 pieces; and another 29 pieces. How many pieces were in all the boxes?

5. John's father worked in a box factory. In 3 days he made 27 boxes. How many boxes was that a day, if he made the same number every day?

6. A milkman started out with 98 pint bottles of cream. He served 84 of them to his customers. How many pints of cream did he bring back?

7. My bookcase has three shelves. On the top shelf are 21 books; on the next shelf 17 books; and on the bottom shelf 19 books. How many books are in the bookcase?

8. Mrs. Waters set 6 hens, putting 12 eggs under each. How many eggs did she use?

9. Harry read a story 72 pages long. If he read 9 pages each evening, how many evenings did it take him to read the story?

10. John had 36 blue counters and 8 pink ones. How many counters did he have?

11. Mr. James had 28 horses and sold  $\frac{1}{4}$  of them. How many horses did he sell?

12. A milkman had four cans of milk in his wagon, containing 24 quarts, 22 quarts, 20 quarts,

and 19 quarts respectively. How many quarts of milk were in the four cans?

13. If a family took 12 quarts a week, how many quarts did they take in 9 weeks?

14. 7 boys have 56 chestnuts. If they have the same number apiece, how many chestnuts has each boy?

15. Since school began 27 days have been sunny and 17 cloudy. How many days have passed?

### Drill

Add the following:

<i>First Day.</i>	4	2	1	3	8	6
	5	3	4	5	7	5
	2	7	8	6	8	9
	<u>+6</u>	<u>+7</u>	<u>+7</u>	<u>+2</u>	<u>+3</u>	<u>+3</u>

<i>Second Day.</i>	4	2	5	5	6	2
	4	7	7	6	3	4
	2	8	9	7	8	9
	<u>+7</u>	<u>+8</u>	<u>+8</u>	<u>+2</u>	<u>+4</u>	<u>+4</u>

<i>Third Day.</i>	51	49	62
	62	89	43
	28	48	89
	<u>+89</u>	<u>+22</u>	<u>+55</u>

<i>Fourth Day.</i>	34	54	52
	76	88	24
	23	59	89
	<u>+92</u>	<u>+12</u>	<u>+66</u>

## PART XII

I and II. Read and write numbers to 600.

III. Solve problems as described below.

It is to be remembered :

1. That problems are to be announced as wholes. Problems should be announced in writing oftener than in Part XI, so that pupils may be prepared to use a text book when Part XII is completed.
2. That the modes of counting are:
  - (a) Additive.
  - (b) Subtractive.
  - (c) Partitive.
3. That the numerical expressions employed are:
  - (a) Additive.
  - (b) Subtractive.
  - (c) Partitive.
  - (d) Multiplicative.
  - (e) Measurement.
4. In all operations the numerical expressions are suggested by imagined counting.

In addition and subtraction the results are obtained by computation.

In multiplication and both cases of division results are obtained by reference to the tables.

IV. 1. Review the addition table and the extensions of groups (a), (b), (c), (d), (e), and (f).

2. Extend groups (g) and (h), as follows:

### GROUP (g)

#### FIRST COMBINATION

7	7	7	7	7	7	7	7	7	7
+3	+13	+23	+33	+43	+53	+63	+73	+83	+93
<hr/> 10	<hr/> 20	<hr/> 30	<hr/> 40	<hr/> 50	<hr/> 60	<hr/> 70	<hr/> 80	<hr/> 90	<hr/> 100

#### SECOND COMBINATION

7	7	7	7	7	7	7	7	7	7
+4	+14	+24	+34	+44	+54	+64	+74	+84	+94
<hr/> 11	<hr/> 21	<hr/> 31	<hr/> 41	<hr/> 51	<hr/> 61	<hr/> 71	<hr/> 81	<hr/> 91	<hr/> 101

#### THIRD COMBINATION

7	7	7	7	7	7	7	7	7	7
+5	+15	+25	+35	+45	+55	+65	+75	+85	+95
<hr/> 12	<hr/> 22	<hr/> 32	<hr/> 42	<hr/> 52	<hr/> 62	<hr/> 72	<hr/> 82	<hr/> 92	<hr/> 102

## GROUP (h)

## FIRST COMBINATION

5	5	5	5	5	5	5	5	5	5
+3	+13	+23	+33	+43	+53	+63	+73	+83	+93
<hr/> 8	<hr/> 18	<hr/> 28	<hr/> 38	<hr/> 48	<hr/> 58	<hr/> 68	<hr/> 78	<hr/> 88	<hr/> 98

## SECOND COMBINATION

6	6	6	6	6	6	6	6	6	6
+3	+13	+23	+33	+43	+53	+63	+73	+83	+93
<hr/> 9	<hr/> 19	<hr/> 29	<hr/> 39	<hr/> 49	<hr/> 59	<hr/> 69	<hr/> 79	<hr/> 89	<hr/> 99

## THIRD COMBINATION

6	6	6	6	6	6	6	6	6	6
+4	+14	+24	+34	+44	+54	+64	+74	+84	+94
<hr/> 10	<hr/> 20	<hr/> 30	<hr/> 40	<hr/> 50	<hr/> 60	<hr/> 70	<hr/> 80	<hr/> 90	<hr/> 100

## V. Difficulties in concrete problems:

1. Artificial units: (a) pints, (b) quarts, (c) gallons, (d) *ounces and pounds* (page 186), (e) *years and months* (page 189), (f) *weeks and days* (page 193).
2. *Comparison of numbers* (page 180).

## PROBLEMS

## FIRST WEEK

1. Show pupils how to solve problems in addition and subtraction, when two numbers are compared, one not being a part of the other.

*Problem.* John has 4 pigeons, and William has 3 more pigeons than John. How many pigeons has William?

**Solution by Counting**

- (1) Take John's pigeons from the supply.
- (2) Take the same number of William's pigeons.
- (3) Take 3 more for William.

Arrange the counters thus:

- (1) o o o o = John's pigeons.
- (2) o o o o and (3) o o o = William's pigeons.

It will be noticed that William's number of pigeons is composed of two parts. One of these parts is not stated directly in the problem, but is said to be equal to all of John's. The other part is the excess of William's over John's. The numerical expression is  $\begin{smallmatrix} 4 \\ +3 \end{smallmatrix}$  in which both numbers represent parts of William's pigeons.

*Second Problem.* Mary has 12 apples and Elizabeth has 7. Mary has how many more apples than Elizabeth?

**Solution by Counting**

- (1) Take Mary's 12 apples from the supply and place them in a row.
- (2) Take Elizabeth's 7 apples from the supply and place them in a row below Mary's.
- (3) Move to the right that portion of Mary's row which extends beyond Elizabeth's and count it, finding it to be 5.



Arrange the counters thus:

(1) oooooooo and (3) ooooo = Mary's apples.

(2) oooooooo = Elizabeth's apples.

The numerical expression is  $\frac{12}{5}$ , in which the 12 represents all of Mary's apples, while the 7 and 5 represent the parts into which Mary's apples are separated, when compared with Elizabeth's.

### Drill

(a) John has 9 tops, which is 2 more than Samuel has. How many tops has Samuel?

(b) Wilson has 16 books, which is 13 less than Thomas has. How many has Thomas?

(c) Harry has 5 pictures. Mary has 9. How many more has Mary than Harry?

(d) How many pictures less than Mary has Harry?

2. There were 243 roses on one hedge and 132 on another. How many more roses were on the first hedge than on the second?

3. In a schoolroom were 32 pupils. They recited in groups of 8. How many such groups were there?

4. Each of 4 tin buckets held 3 quarts of cider. How much cider was there in all the buckets?

5. A fruit grower used 124 gallons of spray on his apple trees, 84 gallons on his pear trees, and 58 gallons on his peach trees. How many gallons did he use altogether?

6. A farmer owns 27 cows. His neighbor has 18 more than he. How many cows has his neighbor?

7. Henry has 8 times as many pigeons as Samuel. Samuel has 4 pigeons. How many has Henry?

8. Mary had 12 photographs taken. Charles had four less than Mary. How many photographs did Charles have taken?

9. There were 18 chickens in one yard and 14 in another. How many chickens in both yards?

10. It took 9 jugs of the same size to hold 18 gallons of cider. How much cider was in each jug?

11. When Martha was 8 years old, her father bought her 48 candles, so that she might place 8 candles in each of her birthday cakes. How many birthday cakes had she?

12. 27 pounds of butter were made into 9 equal rolls. How many pounds were put into each roll?

13. Mr. Smith's water tank holds 165 gallons. Mr. Jones's tank holds 78 gallons more than Mr. Smith's. How much water does Mr. Jones's tank hold?

### Drill

*First Day.* Extensions of group (g).

*Second Day.* Extensions of group (h).

<i>Third Day.</i>	8	1	1	1	3	7
	7	7	7	5	6	6
	<u>+3</u>	<u>+4</u>	<u>+5</u>	<u>+3</u>	<u>+3</u>	<u>+4</u>

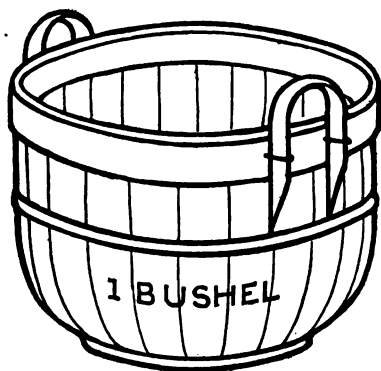
<i>Fourth Day.</i>	4	2	8	2	4	2
	3	4	4	3	3	5
	<u>+6</u>	<u>+7</u>	<u>+6</u>	<u>+5</u>	<u>+7</u>	<u>+7</u>

## SECOND WEEK

## 1. Pecks and bushels counted.

(a) Show pupils a peck measure, and describe an exercise similar to the counting of pints. The presence of the measure will stimulate the imagination

## COUNTING PECKS AND BUSHELS



so that the process will be understood without actually measuring anything.

(b) Show pupils a bushel measure and with it conduct an exercise similar to (a).

2. In one field 46 bushels of wheat were raised. In another field 15 bushels more were raised than in

the first. How many bushels of wheat were raised in the second field?

3. How many pecks of apples in 8 barrels, if each barrel holds 10 pecks?

4. A farmer used each week 6 bushels of bran for cow feed. In how many weeks did he use 42 bushels?

5. There were 97 bushels of corn in a crib. 46 bushels were fed to the horses. How many bushels remained in the crib?

6. There were four rafts of logs in the river. The first raft had 45 logs in it; the second 62 logs; the third 29 logs; and the fourth 78 logs. How many logs in all four rafts?

7. There were 3 water coolers in a waiting room with 6 gallons of water in them. All the coolers held the same quantity of water. How many gallons were in each?

8. 40 pecks of potatoes were put into sacks, 5 pecks in each sack. How many sacks were used?

9. In an ice cream parlor 24 people were seated around 4 tables, the same number at each table. How many were at each table?

10. A grocer delivered 2 gallons of vinegar to each of 7 families. How many gallons did he deliver to them all?

11. A farmer raised 792 bushels of oats. He kept 371 bushels for his own horses. How much did he sell?

12. In a Sunday school there were 15 pupils in one class, 19 in another, 22 in a third, and 27 in a fourth. How many pupils were in the four classes?

13. 2 little girls each paid 9 pins to go to a doll show. How many pins did they both pay?

## Drill

<i>First Day.</i>	5	4	6	5	4	6
	7	8	7	4	8	3
	1	2	9	9	9	8
	<u>+9</u>	<u>+9</u>	<u>+9</u>	<u>+8</u>	<u>+7</u>	<u>+6</u>

<i>Second Day.</i>	8	7	6	7	4	8
	7	4	5	7	2	4
	7	6	1	2	8	8
	<u>+5</u>	<u>+4</u>	<u>+8</u>	<u>+8</u>	<u>+8</u>	<u>+7</u>

<i>Third Day.</i>	3	4	9	8	4	7
	7	6	4	3	2	6
	9	8	7	6	1	2
	<u>+6</u>	<u>+5</u>	<u>+4</u>	<u>+3</u>	<u>+7</u>	<u>+7</u>

<i>Fourth Day.</i>	4	5	8	4	4	9
	6	7	7	5	9	7
	7	7	9	8	7	5
	<u>+7</u>	<u>+6</u>	<u>+5</u>	<u>+4</u>	<u>+3</u>	<u>+3</u>

## THIRD WEEK

## 1. Counting ounces and pounds.

Place on the table a weighing balance, ten or twelve ounce-weights, a bucket of sand, with a scoop, and several paper bags.

Teacher lifts an ounce weight and says, "How heavy do you think this is?"

After various guesses she continues, "It weighs an ounce. All of these (pointing to the ounce weights) weigh an ounce apiece."

She then fills a small paper bag with sand and continues, "How heavy do you think this bag of sand is?"

After various guesses, she says, "Let us see." She then places the bag on one pan of the balance, and on the other pan places ounce weights, one at a time, until a balance is obtained, counting the weights as they are placed in the pan.

## IT WEIGHS SEVEN OUNCES



"It weighs 7 ounces."

Another bag is filled, and is found to weigh 8 ounces. Still another holds 10 ounces.

"How many ounces in the three bags?"

Probably the pupils will find this at once by computation. If not, the counters may be employed.

The pound weight may now be shown the pupils and its relation to the ounce be found with the balance. This, however, is merely incidental, and at this stage is not to be memorized or used in problems.

### Drill

(a) From a bag holding 28 ounces of sugar, 8 ounces were used to make a cake. How many ounces remained in the bag?

(b) 3 large paper bags contained 9 pounds of sugar apiece. How much sugar was in all the bags?

(c) 12 ounces of candy were divided equally among 4 children. How many ounces did each child receive?

2. In the cellar there were 49 pecks of white potatoes, and 37 pecks of sweet potatoes. How many pecks of potatoes were there?

3. A lady put 36 ounces of candy in 4 boxes, so that they all had the same number of ounces. How many ounces of candy were in each box?

4. Our tea can holds 12 ounces of tea. Yesterday mother put 8 ounces into the can and just filled it. How many ounces were already in the can?

5. In a cellar were 54 gallons of cider in 6-gallon kegs. How many kegs were there?

6. A boy picked 6 baskets of cherries. Each basket held 8 pounds. How many pounds of cherries did he pick?

7. A grocer sold to one family 9 ounces of tea; to another 8 ounces; and to a third 14 ounces. How many ounces of tea did the three families buy?

8. George picked 9 quarts of berries. Richard picked 3 quarts less than George. How many quarts did Richard pick?

9. There were 9 pounds of butter in each of 8 butter tubs. How many pounds of butter in all the tubs?

10. A farmer sold 118 sheep. If he had 249 at first, how many were left?

11. An apple grower packed his apples in barrels that held 2 bushels each. How many barrels would he need for 10 bushels?

12. If 8 ounces of silver are used in making a dish, how many such dishes can be made of 32 ounces of silver?

13. One boy weighs 64 pounds. Another boy weighs 19 pounds more than the first. What is the weight of the second boy?

14. A grocer put 60 eggs in 5 small boxes; putting the same number in each box. How many eggs were in each small box?

15.  $18 + 14 + 19 = ?$



## Drill

<i>First Day.</i>	567	284	<i>Third Day.</i>	486	525
	483	306		257	681
	725	514		483	304
	<u>+146</u>	<u>+293</u>		<u>+174</u>	<u>+275</u>

<i>Second Day.</i>	254	107	<i>Fourth Day.</i>	586	259
	810	952		297	187
	196	684		834	263
	<u>+539</u>	<u>+715</u>		<u>+516</u>	<u>+451</u>

## FOURTH WEEK

1. Counting years and months.

Teacher says, "Martha, how old are you?"

"Seven."

Teacher places 7 counters on the table.

"In three more years (placing 3 more counters on the table), how old will Martha be?"

Hands are shown, and the one called on answers.

"Ten."

"Martha, you may write the story on the board."

Martha writes, 7

$$\begin{array}{r} +3 \\ 7 \\ \hline 10 \end{array}$$

"A boy began to go to school when he was 6 years old, and left when he was 14 years old. How many years did he go to school?"

"How old was he when he left?"

"Fourteen."

"Then take 14 counters from the supply."

"How many of these years did he live before he went to school?"

"Six."

"Take 6 of them away from the 14."

"Now how many are left in which he went to school?"

"Eight."

"You may all write the story."

Pupils write, 14

$$\begin{array}{r} 14 \\ - 6 \\ \hline 8 \end{array}$$

"How many months in the year, class?"

If no one knows, teacher answers, "Twelve." The twelve months are named and their names written on the blackboard, thus:

January	April	July	October
February	May	August	November
March	June	September	December

A counter is moved from the supply as each month is named.

"How many months do you have vacation?"

"Two."

Two of the counters are moved from the 12.

"Then how many months are left for school?"

"Ten."

“Write the story.”

Pupils write, 12

$$\begin{array}{r} -2 \\ \hline 10 \end{array}$$

“A lady made 3 trips to Europe. Each trip she was gone 4 months. How long was she gone altogether?”

Pupils write,  $3 \times 4 = 12$ .

2. A pupil went to school 27 months in 3 years. He attended the same length of time each year. How many months a year did he go to school?

3. Mr. St. Clair was in Europe 28 months. This was 17 months longer than Mr. Williams was in Europe. How long was Mr. Williams in Europe?

4. At a party, 9 children were given 4 ounces of candy each. How much candy was used?

5. A boy worked in several stores, 6 months in each. In 24 months, at how many places had he worked?

6. John was born in Salem and lived there 6 years. He then moved to Oregon City and lived there 5 years. He next moved to Portland, where he has lived 4 years. How old is John?

7. A boy picked 9 quarts of berries on Monday, 11 quarts on Tuesday, 12 quarts on Wednesday, and 8 quarts on Thursday. How many quarts did he pick in the four days?

8. 18 pounds of spice filled 3 boxes of the same size. How many pounds were in each box?

9. There are 54 children today in the primary grade of a school. This is 13 more than were present yesterday. How many primary pupils were in school yesterday?

10. 45 pounds of coffee were put up in 5-pound packages. How many packages were there?

11. There were 34 pigeons in one pigeon house. In a second pigeon house there were 15 more pigeons than in the first. How many pigeons were in the second house?

12. Lucy lives 8 blocks from the schoolhouse. She can walk a block in 2 minutes. In how many minutes can she walk to school?

13. A man, who is 24 years old, has spent his life in 3 cities. He has lived the same time in each. How many years has he spent in each city?

### Drill

<i>First Day.</i>	18	56	27	<i>Third Day.</i>	186	285
	21	63	30		295	643
	24	71	40		304	508
	<u>+24</u>	<u>+85</u>	<u>+28</u>		<u>+687</u>	<u>+712</u>

<i>Second Day.</i>	159	386	<i>Fourth Day.</i>	426	464
	807	295		329	375
	236	764		465	224
	<u>+548</u>	<u>+385</u>		<u>+328</u>	<u>+118</u>

## FIFTH WEEK

1. Counting weeks and days.

"How many days in a week?"

"Seven."

Class names them and teacher writes the names on the board, thus:

Sunday

Monday✓

Wednesday✓

Friday✓

Tuesday✓

Thursday✓

Saturday

"On which days do you go to school?"

As the school days are named, teacher checks them as shown above.

"On how many days is there no school?"

"Two."

"You may write the story."

Pupils write, 7

$$\begin{array}{r} -5 \\ \hline 2 \end{array}$$

## Drill

Describe the counting in each of the following problems:

(a) Mary spent 24 weeks visiting 3 eastern cities. If she divided her time equally among them, how long did she visit each?

(b) It takes Louise 3 days to read a book. How many such books can she read in 21 days?

2. Last year we planted 17 rosebushes, but 8 of them have not yet bloomed. How many have bloomed?

3. Martin left home with his father and traveled 13 weeks. They then spent 9 weeks on a farm. Martin then went to boarding school for 24 weeks. After that he went home again. How many weeks had he been away?

4. Mr. Brewster's cows each gave 5 quarts at a milking. He got altogether 40 quarts at a milking. How many cows had he?

5. There are 96 days in the present school term, and 45 days are gone. How much of the term remains?

6. 42 pounds of sugar were put into sacks, 6 pounds in each sack. How many sacks were used?

7. 35 pints of root beer were put into jugs holding 5 pints each. How many jugs were used?

8. A little girl had 14 ounces of candy. She gave some of it to her friends and had left 5 ounces. How much did she give away?

9. A party of boys encamped in the woods 18 days. During that time they were in 3 camps, spending the same time in each. How many days were they in each camp?

10. John went to the country to visit 3 cousins. He spent 7 days with each. How long did he stay in the country?

11. The policeman in our neighborhood is changed

every 3 months. In 12 months how many different policemen have we had?

12. A hotel has 18 rooms on the first floor, 21 on the second, 24 on the third, and 24 on the fourth. How many rooms in the hotel?

13. A butcher took a piece of meat weighing 20 pounds and divided it equally among 5 families. How many pounds did each family receive?

### Drill

<i>First Day.</i>	584	320	<i>Third Day.</i>	187	436
	632	756		946	700
	789	483		783	587
	<u>+451</u>	<u>+975</u>		<u>+591</u>	<u>+294</u>

<i>Second Day.</i>	274	400	<i>Fourth Day.</i>	562	453
	361	107		748	279
	248	204		357	856
	<u>+563</u>	<u>+508</u>		<u>+681</u>	<u>+724</u>

### SIXTH WEEK

1. Mary had 124 needles. Josephine had 13 needles less than Mary. How many needles had Josephine?

2. The mason brought 4 baskets of lime to make mortar. In each basket were 5 pecks. How many pecks of lime did he bring?

3. A lady used 56 ounces of flour to bake 8 loaves of bread of the same weight. How many ounces of flour were in each loaf?

4. John weighs 56 pounds; William 63 pounds; and Henry 97 pounds. How much do the three boys weigh?

5. When Harry went to visit his uncle, he had to ride 38 miles in an automobile. When he had ridden 22 miles how far had he yet to go?

6. How far had Harry ridden when he had only 8 miles farther to go?

7. A hunter bought 6 pounds of meat for each of his 4 dogs. How much meat did he buy?

8. Samuel had 39 cherries, and ate 27 of them. How many had he left?

9. 8 boxes contained 9 tops apiece. How many tops were in all of them?

10. A logger has 42 oxen which he uses to drag logs to the creek. If there are 6 oxen in a team, how many teams has he?

11. How many quarts of cider in 5 jugs, if each jug holds 3 quarts?

12. I read 28 pages of my new book yesterday, and 14 pages this morning; I have 90 pages yet to read. How many pages are there in my book?

13. The A class had 59 pupils, and the B class 47. How many more pupils were there in the A class than in the B class?

14. On a Christmas tree were 8 sacks of candy.



There were 3 ounces in each sack. How many ounces of candy were on the tree?

15. A grocer had some small butter tubs which held 9 pounds each. How many were needed to hold 36 pounds?

### Drill

<i>First Day.</i>	400	374	<i>Third Day.</i>	427	459
	605	269		685	683
	780	185		437	725
	<u>+509</u>	<u>+845</u>		<u>+291</u>	<u>+486</u>
<i>Second Day.</i>	567	218	<i>Fourth Day.</i>	591	427
	890	497		274	638
	123	653		863	597
	<u>+456</u>	<u>+964</u>		<u>+759</u>	<u>+284</u>

# TOPICAL INDEX

## NOTATION AND NUMERATION

### I. To teach counting, page 6.

#### 1. Points requiring attention:

- (a) New number names, page 6.
- (b) Learning the scale, page 6.
- (c) Errors in counting, pages 6 and 9.
- (d) The two general modes of counting, pages 7 and 8.
- (e) Grading of material upon which counting is practiced, page 8.
- (f) Application of (a) and (e), pages 9 and 10.

### II. Writing Numbers.

#### 1. Points requiring attention:

- (a) Use of blackboard in daily work, page 10.
- (b) Grouping of objects counted, pages 10 and 11.

### III. Solution of Problems by Counting.

- (a) Both of the general modes of counting employed, pages 11 and 12.
- (b) Stages of objective solution, pages 12 and 13.

## ADDITION

- (a) Additive counting, first stage, real objects, page 13.
- (b) Additive counting, second stage, pictures, page 15.
- (c) Additive counting, third stage, i. e., on number table with counters, page 19.
- (d) Numerical expression of additive counting, page 20.

## SUBTRACTION

- (a) Subtractive counting, first stage, real objects, page 13.
- (b) Subtractive counting, second stage, pictures, page 16.
- (c) Subtractive counting, third stage, on number table with counters, page 21.
- (d) Numerical expression of subtractive counting, pages 22 and 23.

**DIVISION****I. Partition:**

- (a) Partitive counting, first stage, real objects, page 14.
- (b) Partitive counting, second stage, pictures, page 16.
- (c) Partitive counting, third stage, with counters on the number table, page 24.
- (d) Numerical expression of partitive counting, pages 25 and 26.

**II. Measurement:**

- (a) Subtractive counting, continued taking away of equal parts, page 31.
- (b) The numerical expression of such counting.

**MULTIPLICATION**

- (a) Additive counting of equal parts, page 28.
- (b) The multiplicative expression, pages 28 and 29.
- (c) The arithmetical notion of multiplication, note pages 29 and 30.

**ANNOUNCEMENT OF PROBLEMS**

- (a) Step by step, page 26.
- (b) As wholes, page 26.

**DRILLS ON NUMERICAL EXPRESSIONS**

See pages 34 and 35.

**WORK AT DESK**

See page 36.

# Numbers Step by Step

By FRANK RIGLER

Author of "A Primer of Number"

The features that distinguish this book from other elementary arithmetics, and which in the experience of the author make it more teachable, are:

**FIRST.** The general arrangement of the book is topical in the truest sense. The difficulties are arranged in what is believed to be the order of "least resistance." Each difficulty is then made the subject of a lesson.

**SECOND.** The fact, that all concrete problems may be solved by counting instead of by computation, is used to determine, in a given problem, which of the computative processes is to be employed.

**THIRD.** The multiplication table is used for reference purposes before it is learned, thus furnishing an adequate motive for the stupendous task of memorizing it.

**FOURTH.** Artificial units are introduced with the utmost care. Such attributes as weight, capacity, and time, do not naturally suggest plurality. The assumption that number can be artificially applied to them by young children without careful instruction is erroneous. The practice based on this assumption is sure to dull the number perception of pupils.

**FIFTH.** The computative process of long division is so treated that its three difficulties are met and mastered one at a time.

**Cloth. Introduction and Mailing Price, Forty Cents.**

O. P. Barnes, Publisher, Chicago.



